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## Monterey, California



# THESIS

STATE-SPACE ANALYSIS OF  
THE TRIDENT II MARK 6 GYRO

by

David L. Krueger

March 1986

Thesis Advisor:

D. J. Collins

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State-Space Analysis  
Of  
the Trident II Mark 6 Gyro

by

David L. Krueger  
Lieutenant, United States Navy  
B.S., United States Naval Academy, 1979

Submitted in partial fulfillment of the  
requirements for the degree of

MASTER OF SCIENCE IN AERONAUTICAL ENGINEERING

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March 1986

## ABSTRACT

This thesis conducts a state-space analysis of the Mark 6 gyro used in the Trident II missile system. Optimal control techniques, utilizing a region controller, have been employed in designing a gyro controller. The response of the system to missile motion is examined and compared to the system response using the current control laws. It is shown that an optimal region controller reduces the gyro error in the system while effectively avoiding gimbal lock, a situation which exists when gyro orientation prevents missile motion from being isolated in an orthogonal three axis coordinate system. Recommendations for the application of additional modern control techniques are included.



## TABLE OF CONTENTS

I.	INTRODUCTION. . . . .	8
II.	FUNCTION OF THE GYRO CONTROLLER . . . . .	10
	A. THE ALL-ATTITUDE GIMBAL ASSEMBLY . . . . .	10
	1. Gimbal Lock . . . . .	10
	2. Missile to Stable Member Motion Coupling . . . . .	10
III.	MARK 6 GYRO MODEL . . . . .	13
	A. BLOCK DIAGRAM . . . . .	13
	1. Plant Diagram . . . . .	13
	2. Gyro Controller . . . . .	17
IV.	STATE-SPACE MODEL . . . . .	20
	A. BLOCK DIAGRAM . . . . .	20
	1. Plant Model . . . . .	20
	2. Assumptions and Simplifications . . . . .	20
	3. State-Space Equations . . . . .	20
V.	OPTIMIZATION TECHNIQUE . . . . .	24
	A. OPTIMAL REGULATOR . . . . .	24
	1. Linear Quadratic Problem . . . . .	24
	2. Choice of Weighting Matrices . . . . .	26
	3. Non-zero Steady State Outputs . . . . .	26
	B. MARK 6 GYRO OPTIMAL CONTROLLER . . . . .	27
	1. Region Controller . . . . .	28
VI.	SYSTEM SIMULATION . . . . .	30
	A. SIMULATION MODEL . . . . .	30
	B. WEIGHTING MATRICES . . . . .	30
	1. Torque Limits . . . . .	30
	2. State Output Weighting Matrix $R_1$ . . . . .	30

3.	Control Input Weighting Matrix $R_2$	34
C.	TEST PARAMETERS	35
1.	Initial Conditions	35
2.	Missile Motion	36
D.	TEST RESULTS	36
VII.	CONCLUSIONS AND RECOMMENDATIONS	42
A.	CONCLUSIONS	42
1.	Optimal Design Techniques	42
B.	RECOMMENDATIONS	43
1.	Improving System Performance	43
APPENDIX A:	DEFINITIONS	44
APPENDIX B:	FOUR GIMBAL GYRO STATE AND ERROR EQUATION DEVELOPMENT	45
1.	FOUR GIMBAL IMU MODEL	45
1.	Assumptions and Approximations	45
2.	Definitions	45
3.	Symbols	46
4.	Transformation of missile motion to inertial member	46
APPENDIX C:	OPTIMAL CONTROL PROGRAM LISTING	51
APPENDIX D:	TABULATED DATA	66
	LIST OF REFERENCES	68
	INITIAL DISTRIBUTION LIST	69

## LIST OF TABLES

I	STATE AND CONTROL VECTORS . . . . .	22
II	OUTPUT STATE VECTOR MAXIMUM VALUES . . . . .	33
III	VARIABLE DEFINITIONS . . . . .	44

## LIST OF FIGURES

2.1	Four Gimbal Gyro Model . . . . .	11
2.2	Four Gimbal Gyro Coordinate Systems . . . . .	12
3.1	Mark 6 Gyro Model (Inertial Mode) . . . . .	14
3.2	Resolution of Outer and Middle Gimbal Inertias . .	15
3.3	Gimbal Lock Avoidance Switching Line . . . . .	17
3.4	Mark 6 Gyro Control Law . . . . .	18
4.1	State-Space Model of the Mark 6 Gyro . . . . .	21
4.2	Plant and Control Matrices for State-Space Model . . . . .	23
5.1	Optimal Gain Feedback Block Diagram . . . . .	27
5.2	Region Controller Segments . . . . .	29
5.3	Algorithm for the Mark 6 Gyro Optimal Regulator . .	29
6.1	State-Space Simulation Model of the Mark 6 Gyro . .	31
6.2	Plant Matrix for the State-Space Simulation Model . . . . .	32
6.3	Control Matrix for the State Space Simulation Model . . . . .	33
6.4	U and V gyro Errors for the State-Space Model . . .	37
6.5	UGSGF Errors - Draper Lab and State-Space Models . . . . .	39
6.6	VGSGF Errors - Draper Lab and State-Space Models . . . . .	40
6.7	Inner and Middle Gimbal Angles . . . . .	41
B.1	Four Gimbal Gyro Block Diagram . . . . .	50



## I. INTRODUCTION

A need for accurate, self-contained navigation systems has existed since the early days of aircraft design. The most common of these systems uses accelerometers to determine body motion. The output signal is integrated once to obtain velocity and again to obtain position.

In order for the navigation system to accurately determine body motion the sensors must be mounted on a platform fixed in a known coordinate system. In ballistic missile guidance systems, gyros are used to fix a platform in inertial space. The accelerometers are mounted on this platform. The purpose of the gyro controller, then, is to generate the gyro torque motor commands necessary to maintain the stable platform fixed in inertial space.

Since 1957, the Charles Stark Draper Laboratory in Cambridge, Massachusetts has designed the gyros for the Navy's Polaris, Poseidon and Trident missiles. Because of the complex, highly non-linear nature of the control system, this design process has been a tedious, iterative affair. Using state-space representation of systems, computer-aided optimal control design techniques greatly simplify the multi-input, multi-output non-linear time varying problem. The goal of this thesis is the development of a state-space model of the Trident II gyro system, the Mark 6, and the application of modern control theory to improve system performance.

A state-space model of the system has been developed and is compared to the current gyro model. Transient response of both systems to missile motion is examined. Modern control techniques using optimal feedback gains are employed to minimize gyro errors and improve system response. The choice of optimal feedback gains is made based on a region

controller which selects the gains dependent upon the particular gimbal angles. Finally, suggestions are made for employing modern control techniques to further optimize system performance.

## II. FUNCTION OF THE GYRO CONTROLLER

### A. THE ALL-ATTITUDE GIMBAL ASSEMBLY

The inertial navigation system for a ballistic missile must accurately determine missile motion in three orthogonal planes. Missile motion is resolved by accelerometers mounted on a platform fixed in inertial space. A three dimensional coordinate system is initialized before missile launch, and remains fixed through out the flight. The function of the gyro controller is to keep the stable platform fixed throughout the flight.

The Mark 6 gyro consists of four concentric gimbals as shown in Figure 2.1 [Ref. 1: p. 2]. The innermost gimbal is the stable platform, housing the accelerometers. As the missile experiences pitch, roll and yaw a torque is applied to the gimbals in such a manner as to isolate the stable member from the motion.

#### 1. Gimbal Lock

In addition to maintaining a fixed inertial reference, the gyro controller must prevent "gimbal lock". This occurs when the gimbals are co-planar and it becomes impossible to isolate missile motion on three orthogonal axes. Controlling the gyro errors is relatively straightforward, while preventing gimbal lock is more complex. Since the gimbal angles can assume any value during missile flight, the problem becomes non-linear and difficult to solve. The technique developed by Draper Laboratory for preventing gimbal lock will be employed in the design of the state-space system model. This technique is more fully described in the next chapter.

#### 2. Missile to Stable Member Motion Coupling

The four gimbal IMU model is shown in Figure 2.2 [Ref. 2: p. 2]. Missile, or base motion occurs in the

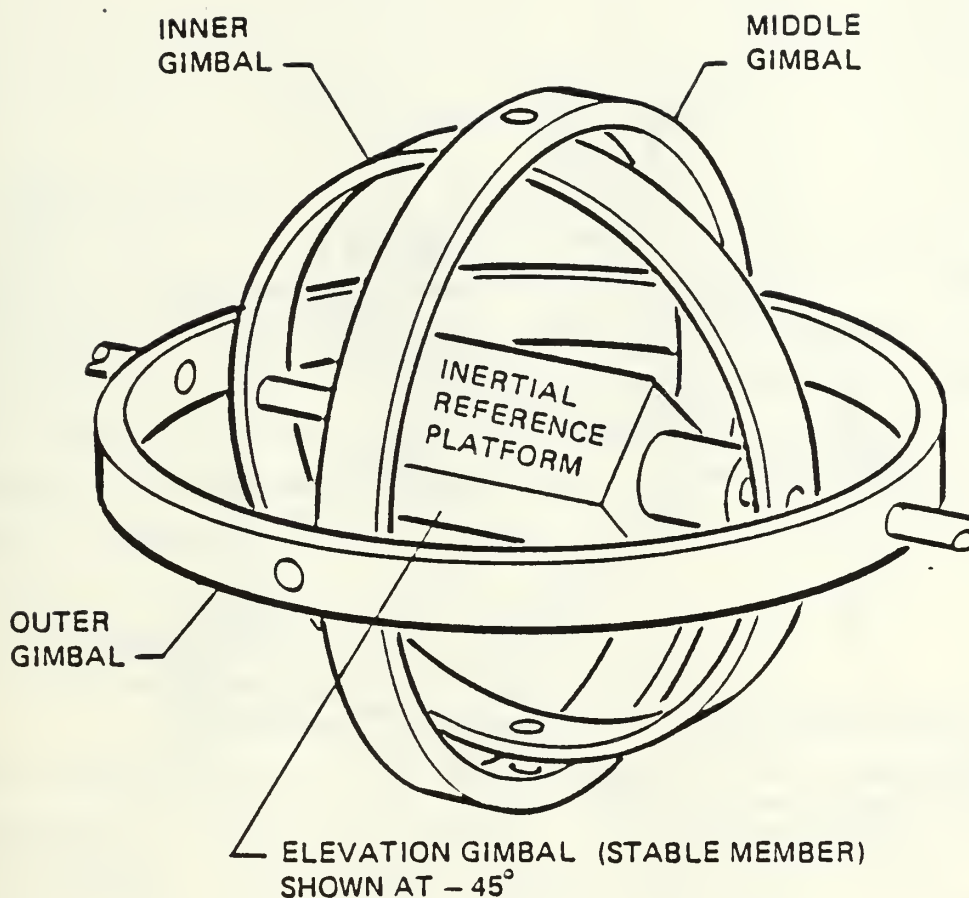


Figure 2.1 Four Gimbal Gyro Model.

$i_5, j_5, k_5$  coordinate system. This motion is coupled to the stable member coordinate system  $i_1, j_1, k_1$  by the outer, middle and inner gimbals. The system accuracy is measured by the gyro error of the stable member in the  $i_1, j_1, k_1$  space. Appendix B contains the detailed transformation of missile motion in the base frame of reference to motion in the inertial frame.



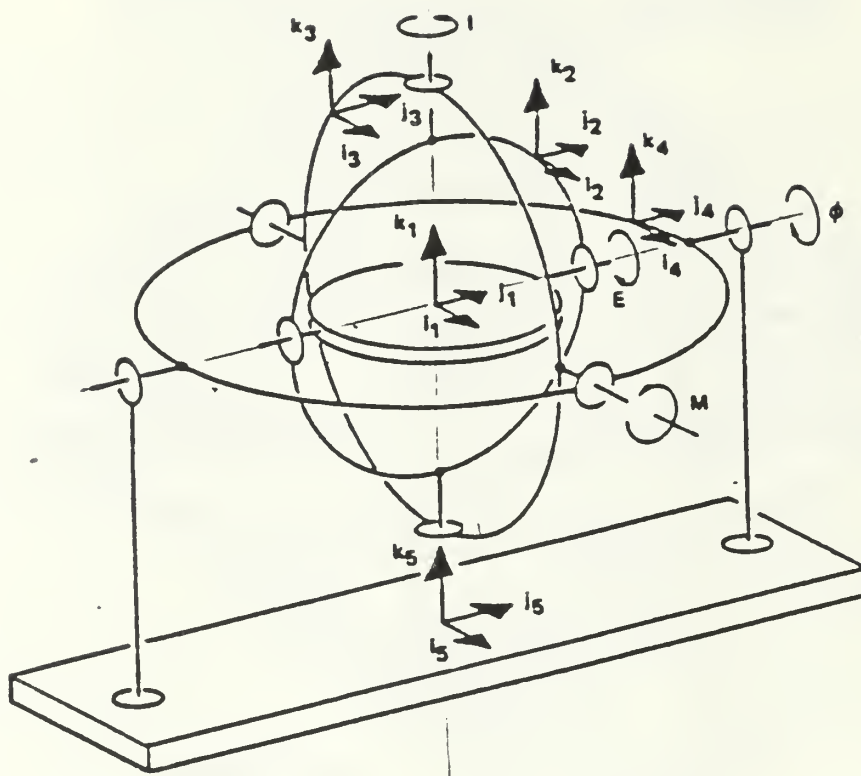


Figure 2.2 Four Gimbal Gyro Coordinate Systems.

### III. MARK 6 GYRO MODEL

#### A. BLOCK DIAGRAM

The Mark 6 Gyro model presently being used by Draper Laboratory is shown in Figure 3.1 [Ref. 2: p. 8]. Table III in Appendix A contains a definition of abbreviations used in this thesis. The torque motor commands are the inputs to the system, with the gyro errors and gimbal angles as the output. The diagram illustrates the cross-coupling between gimbals represented by the input equations shown in Figure B.1.

##### 1. Plant Diagram

###### a. Signal Conversions

Figure 3.1 contains the electrical and mechanical realizations of the Mark 6 gyro. The input signal is expressed in terms of quanta, with one quanta equal to 0.0735 volts. The torque command is expressed in ounce-inches. The gimbal angles, given in radians, are converted to quanta for input to the feedback controller.

###### b. RGyro

Also shown on the diagram is the RGyro. This is not a part of the plant. The RGyro defines an error associated with the roll axis. In earlier three gimbal gyros, the roll axis of the missile did not coincide with any of the gimbal axes. The four gimbal Mark 6 gyro aligns the roll axis with the outer gimbal axis, thus eliminating the need for the RGyro as an indication of gyro error. The output is still provided for ease in comparing current model errors with previous systems.

###### c. Gimbal Inertias

The gimbal inertias are represented in Figure 3.1 as  $J_S$ ,  $J_I$ ,  $J_M$  and  $J_O$  for the stable member and inner, middle and outer gimbals respectively. The outer and middle

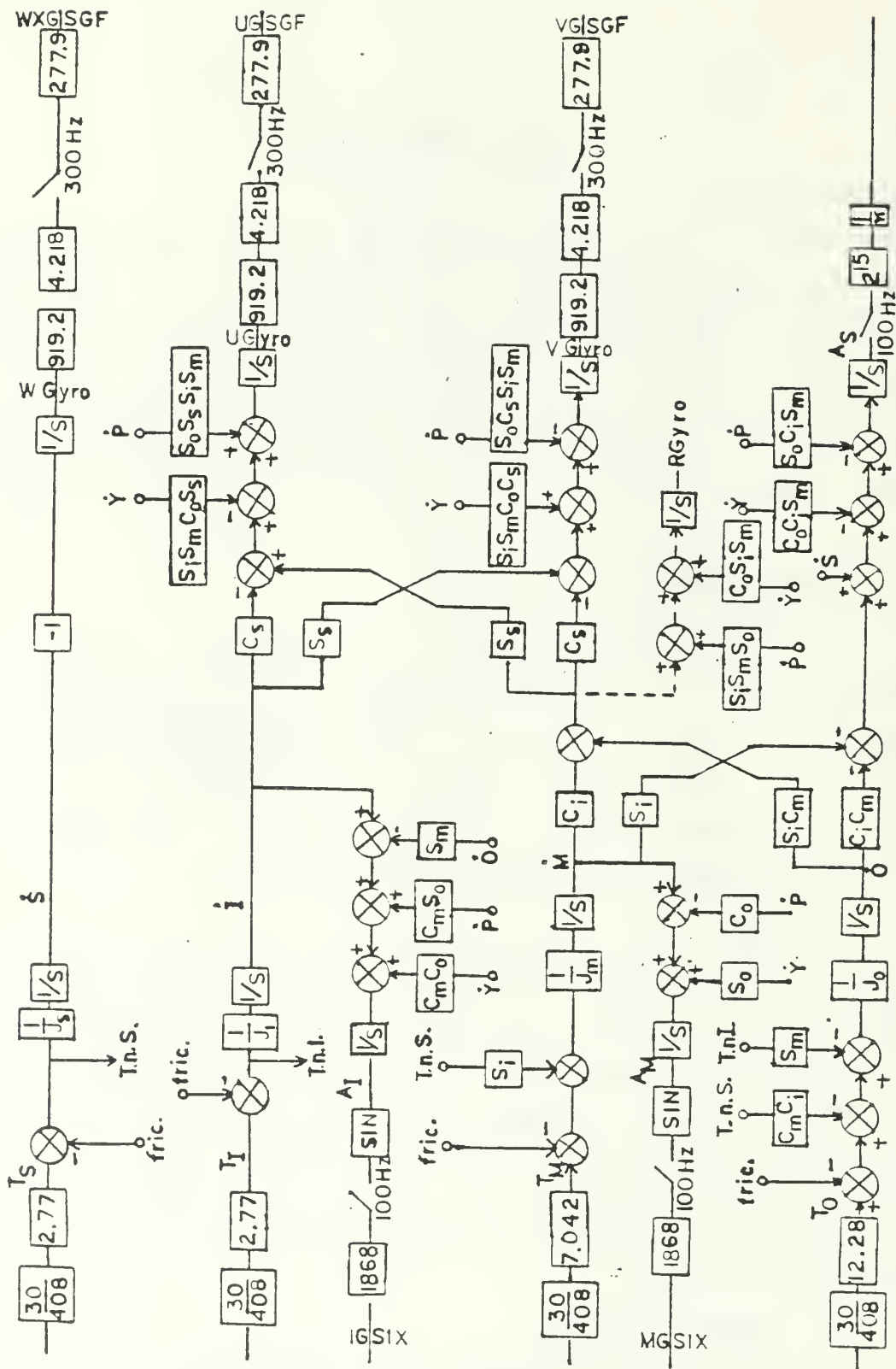
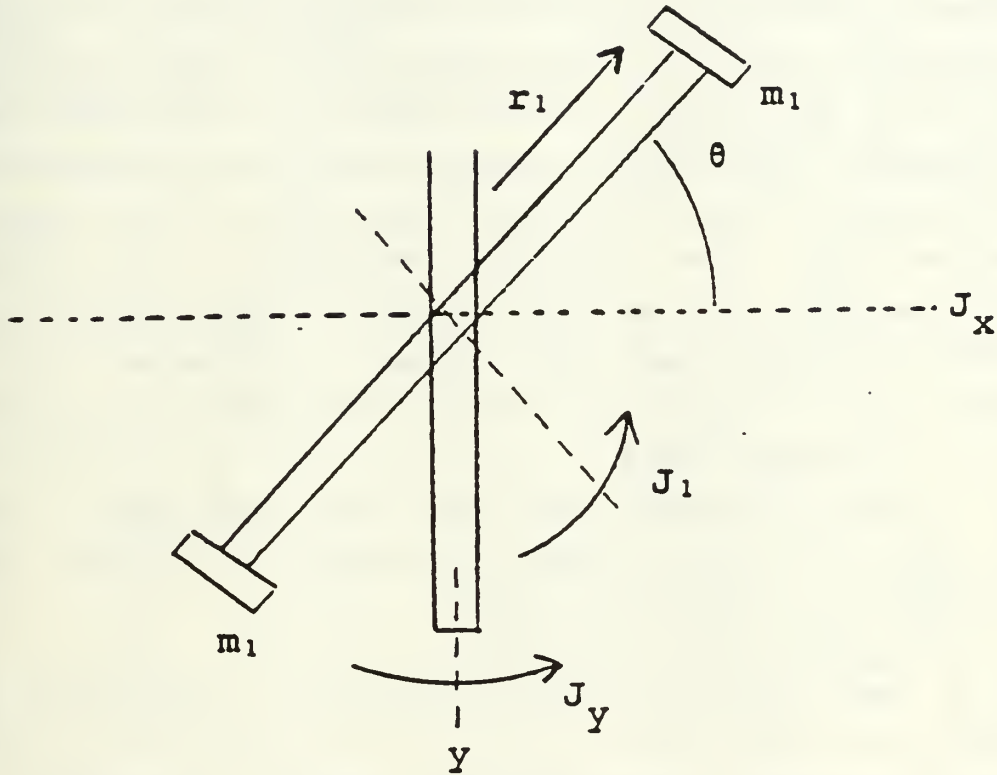


Figure 3.1 Mark 6 Gyro Model (Inertial Mode).

gimbal inertias change with time as a function of the inner gimbal angle. Figure 3.2 shows how the inertias are resolved [Ref. 2: p. 6].



$$J_1 = r_1^2 m_1 \cos^2 \theta$$

resolving along the x and y axes:

$$J_x = I_y^2 m_1 = r_1^2 m_1 \sin^2 \theta = J_o \sin^2 \theta$$

$$J_y = I_x^2 m_1 = r_1^2 m_1 \cos^2 \theta = J_o \cos^2 \theta$$

for the four gimbal model:

$$J_M = J_M + J_I + J_S C_I^2$$

$$J_O = J_O + J_M + J_I C_M^2 + J_S C_M^2 S_I^2$$

Figure 3.2 Resolution of Outer and Middle Gimbal Inertias.



#### d. Stable Element

The stable member is represented by the upper flow path in Figure 3.1. There are no external inputs to the stable member, with only friction affecting the loop. The stable element is controlled by its own feedback loop and is not influenced by the other system signals. Signals from the stable loop do cross-couple to the other gimbals, however, with the torque command coupling to the middle and outer gimbals. The inertial rate  $S$  serves as an input to the stable member angle, shown at the bottom of the figure. This angle could also be shown connected to the stable element flow path, with an input from the outer gimbal inertial rate. Because the stable member is affected only by friction, it has not been included in the state-space model.

#### e. Sampling Rates

Two different sampling rates are used in the system. The U, V and W gyros are sampled at 300 Hz. The gimbal angles are sampled at 100 Hz.

#### f. Gimbal Lock Avoidance

The proximity of the system to gimbal lock can be measured by observing the sines of the inner and middle angles. The product  $\text{SIN}(A_I) * \text{SIN}(A_M)$  gives an indication of how close the system is to gimbal lock, with gimbal lock occurring when the product equals one. The goal of gimbal lock avoidance is to drive the product of the sines to zero. This is accomplished by setting either the inner or middle angle to 0 or 180 degrees.

Only one angle need be controlled at a time. Figure 3.3 shows the regions where control would switch from one angle to another. This is based on the smallest of the absolute values of the sines.

The missile motion input to the U and V gyros has a gain factor of  $\text{SIN}(A_M) * \text{SIN}(A_I)$ , as seen in Figure 3.1. Thus, in addition to avoiding gimbal lock, nulling the

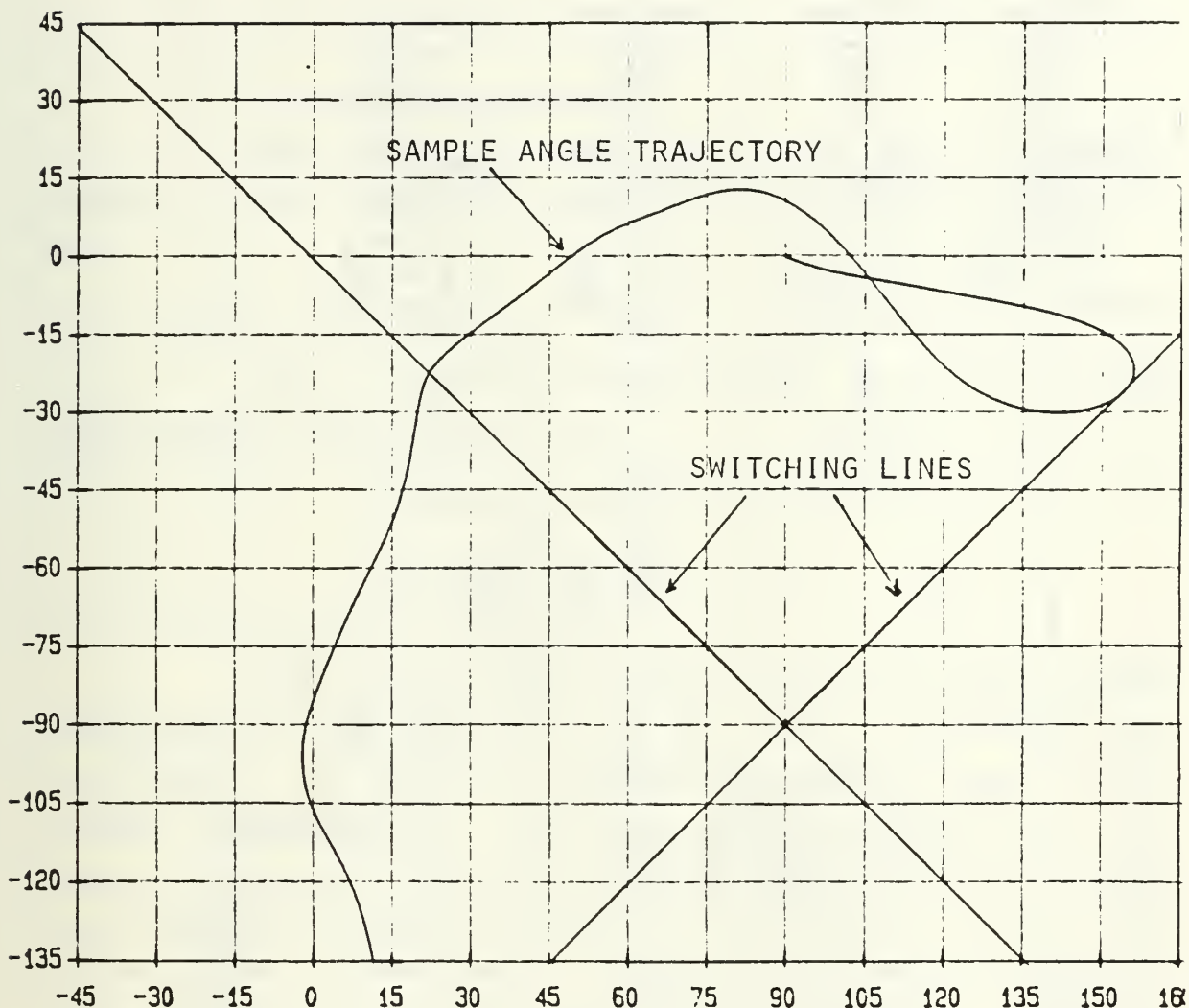


Figure 3.3 Gimbal Lock Avoidance Switching Line.

product of the sines of the inner and middle gimbal angles also minimizes the effect missile motion has on gyro error.

## 2. Gyro Controller

### a. Control Laws

Figure 3.4 shows the controller for the Mark 6 gyro [Ref. 3: p. 115]. The six inputs to the controller are the three signals generated from the U,V and W gyros, the stable element gimbal angle and the sines of the inner and middle gimbal angles. The four outputs are the torque motor command signals.

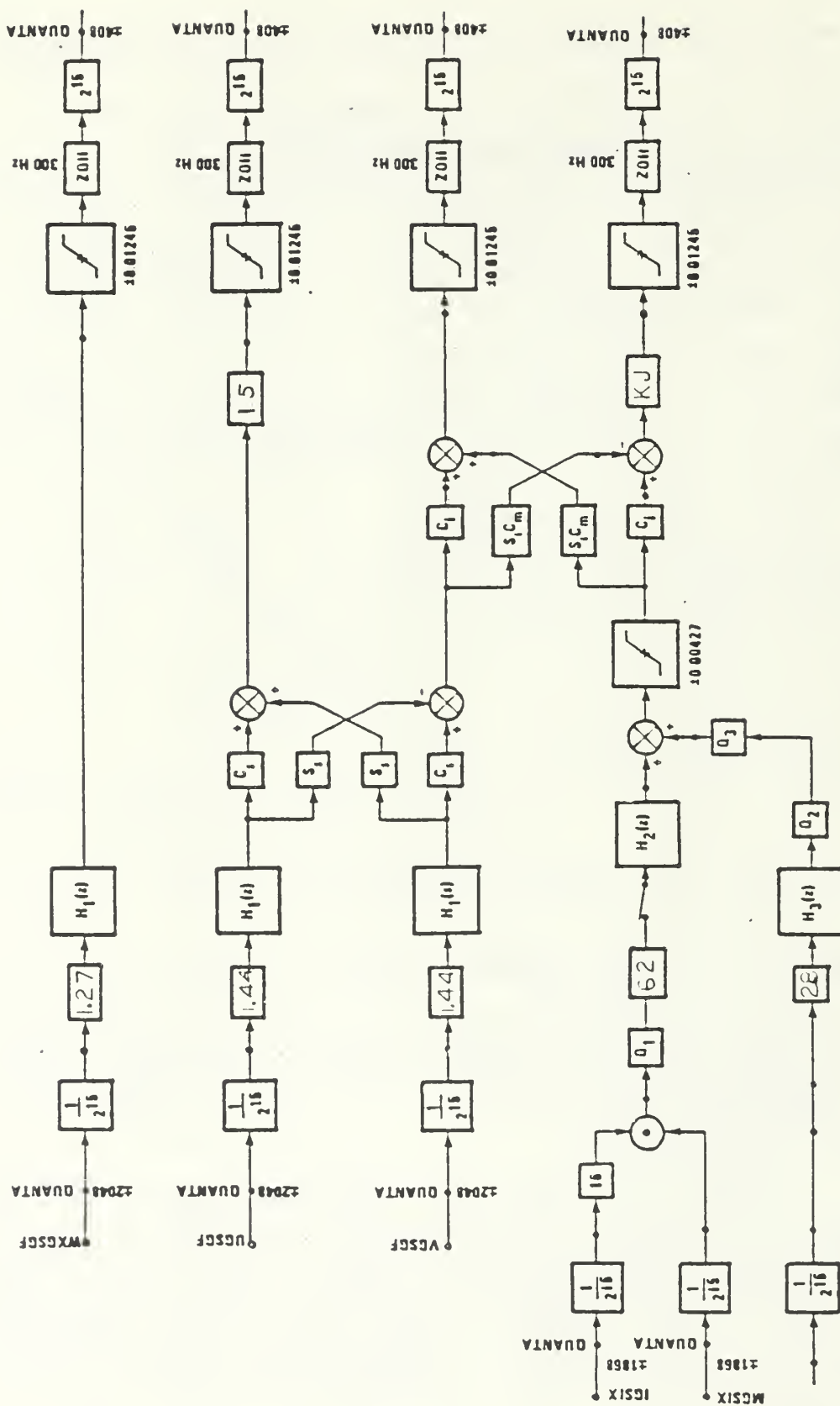


Figure 3.4 Mark 6 Gyro Control Law.

### b. Transfer Functions

The transfer function  $H_1(z)$  in the stable element, inner and middle gimbal feedback paths is defined as:

$$H_1(z) = (z - .932)^3(z - .80955)/(z - 1)^2(z - .6186)(z - .17474)$$

The transfer functions  $H_2(z)$  and  $H_3(z)$  in the outer gimbal control loop are:

$$H_2(z) = (z - .9778)/(z + .3827)$$

$$H_3(z) = (z - 1)/z$$

### c. Redundant Torque

The outer gimbal control loop has three inputs controlled by switches  $Q_1, Q_2$  and  $Q_3$ . Two of the inputs are the  $\text{SIN}(A_M)$  and  $\text{SIN}(A_I)$  signals. The third input is the stable element angle. The two signals generated by this loop are necessary when the product  $\text{SIN}(A_M) * \text{SIN}(A_I)$  becomes very small and does not produce a large enough error signal to drive the outer gimbal.  $Q_1$  and  $Q_3$  are switches with values of +1 and -1 depending on gimbal geometry. Switch  $Q_2$  is closed for values of  $\text{SIN}(A_M) * \text{SIN}(A_I)$  less than 1/8, and open for values greater than 1/8.

### d. Scaling Factors

Gain blocks with  $2^{15}$  and  $1/2^{15}$  scaling factors are included in the control laws. These gain factors are applied in order to facilitate software processing of the feedback signals only.

## IV. STATE-SPACE MODEL

### A. BLOCK DIAGRAM

#### 1. Plant Model

The state-space model of the Mark 6 Gyro is shown in Figure 4.1. Where appropriate, the gains from Figure 3.1 have been combined into a single block. Only the inner, middle and outer gimbals have been modeled. Eight states are identified on the figure.

#### 2. Assumptions and Simplifications

A number of assumptions and simplifications have been made in order to develop the state-space model:

- (a) there is no friction (this eliminates the stable element loop)
- (b) all states are available for feedback
- (c) the inner and middle angles are fed back directly, not the sines of the angles
- (d) there are no limiters in the system
- (e) the entire system is sampled at 300 Hz

#### 3. State-Space Equations

The state-space equations of the system are in the form:

$$\dot{x}(t) = A(t)x(t) + B(t)u(t)$$

The vectors  $x(t)$  and  $u(t)$  are described in Table I.  $A(t)$  and  $B(t)$  are the plant and control matrices, respectively. The matrices for this system are shown in Figure 4.2.



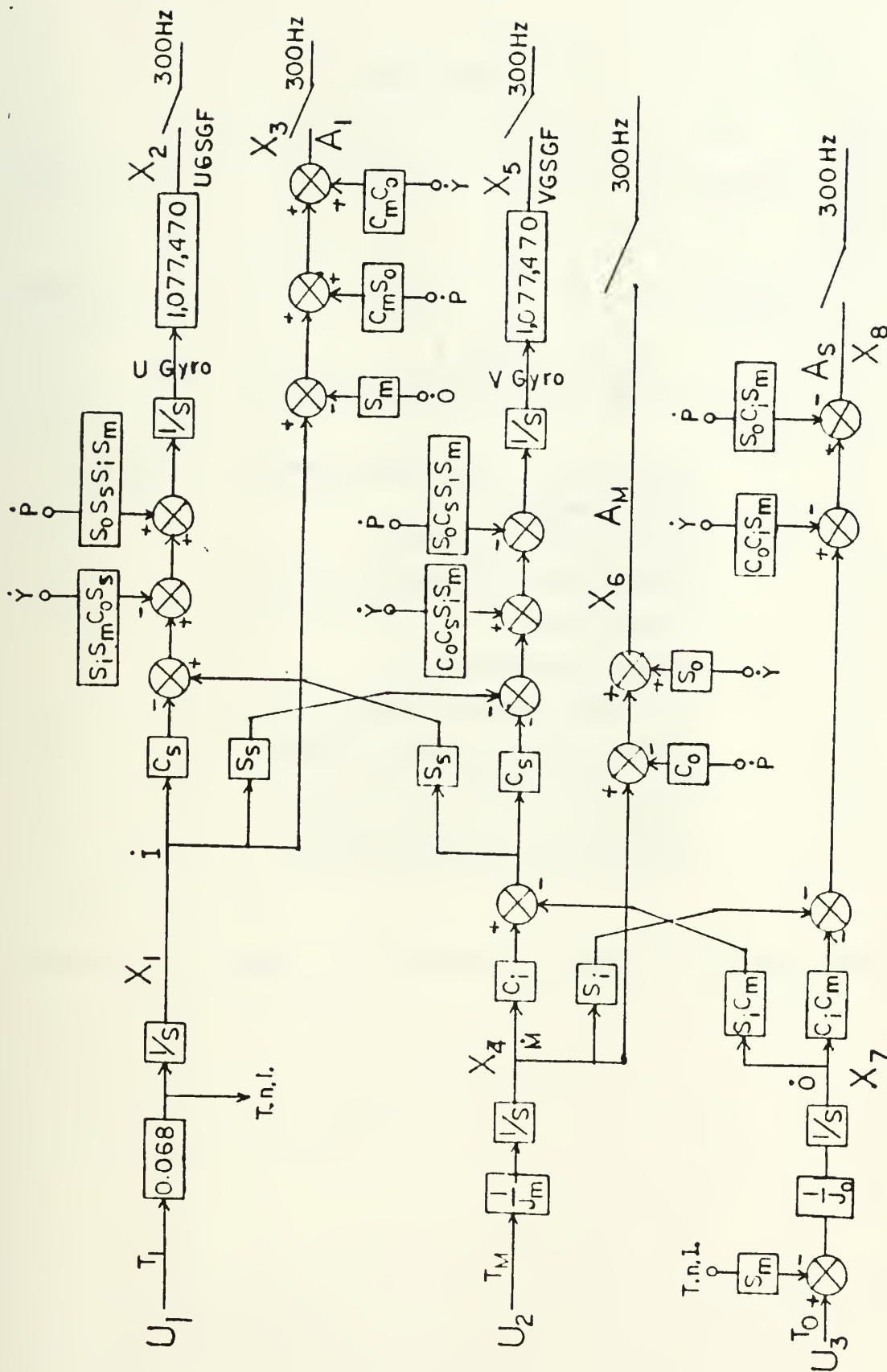


Figure 4.1 State-Space Model of the Mark 6 Gyro.

TABLE I  
STATE AND CONTROL VECTORS

$x_1$	inner gimbal inertial rate I
$x_2$	U Gyro error
$x_3$	inner gimbal angle $A_M$
$x_4$	middle gimbal inertial rate M
$x_5$	V Gyro error
$x_6$	middle gimbal angle
$x_7$	outer gimbal inertial rate O
$x_8$	stable element gimbal angle
$u_1$	inner gimbal torque signal
$u_2$	middle gimbal torque signal
$u_3$	outer gimbal torque signal

Plant Matrix

$$\begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -KC_S & 0 & 0 & KC_I S_S & 0 & 0 & -KS_I C_M S_S & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & -S_M \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ KS_S & 0 & 0 & -KC_I C_S & 0 & 0 & KC_S S_I C_M & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -S_I & 0 & 0 & -C_I C_M & 0 \end{bmatrix}$$

where  $K = 1,077,470$

Control Matrix

$$\begin{bmatrix} 0.068 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 1/J_M & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ -S_M/J_O & 0 & 1/J_O \\ 0 & 0 & 0 \end{bmatrix}$$

Figure 4.2 Plant and Control Matrices for State-Space Model.

## V. OPTIMIZATION TECHNIQUE

### A. OPTIMAL REGULATOR

#### 1. Linear Quadratic Problem

The Mark 6 gyro model can be represented by the state differential equation

$$\dot{x}(t) = A(t)x(t) + B(t)u(t)$$

The goal of the optimization technique is to drive the system to the desired state as quickly as possible, while using only practical input amplitudes. The quadratic integral criterion can be used to express how rapid an initial state is driven to the final state [Ref. 4: p. 201]. This criterion can be expressed as:

$$J = \int x^T(t)R_3(t)x(t)dt$$

$R_3(t)$  is an appropriate weighting matrix chosen to minimize the output state vector. The criterion as represented above would require very large input amplitudes, usually unattainable in a practical system. Including the input in the quadratic integral yields

$$J = \int [x^T(t)R_3(t)x(t) + u^T(t)R_2(t)u(t)]dt$$

By proper selection of  $R_2$  and  $R_3$ , the output vector can be minimized using realizable input amplitudes.

Since only the controlled variables are of interest, the cost equation can be written in terms of the output vector  $z(t)$ , given as:

$$z(t) = D(t)x(t)$$

A weighting matrix  $R_1$  can now be defined as:

$$R_1(t) = D^T(t)R_3(t)D(t)$$

$R_1(t)$  is a square, positive semi-definite symmetric matrix having the same order as the number of controlled variables of the system. The selection of the specific values for  $R_1$  and  $R_2$  are discussed in the next section.

The inputs for the Mark 6 gyro are the signals to the torque motors. These signals are generated solely as a function of gyro error and gimbal angular position. In the optimal regulator problem, the inputs can be expressed in terms of an optimal feedback gain matrix and the states.

$$u(t) = -Fx(t)$$

The state differential equation can then be written as:

$$\dot{x}(t) = A(t)x(t) + B(t)(-Fx(t)) + B(t)u_o(t)$$

When there are no external inputs,  $u_o(t)=0$ . Combining terms yields the following:

$$\dot{x}(t) = [A(t)-B(t)F(t)]x(t)$$

In the linear quadratic problem, the control law can be expressed as:

$$u(t) = -R_2^{-1}B^TPx(t)$$

The optimal feedback gain matrix now becomes:

$$F = R^{-1}B^TP$$

The matrix  $P$  is found by solving the Riccati equation:

$$PA + A^TP - PBR_2^{-1}B^TP + R_1 = 0$$

This is the solution for the steady-state optimal feedback gains, and is the method used to develop the feedback gain matrices used to control the Mark 6 gyro.



## 2. Choice of Weighting Matrices

The weighting matrices  $R_1$  and  $R_2$  were chosen based on the maximum values that the output vector and inputs could be expected to reach. When the weighting matrices have values only on the diagonal, the matrix product  $x^T R x$  becomes  $x_i^2 / R_{ii}$  for each of the values of  $x$ .  $R_{ii}$  is then set equal to the inverse of the maximum expected value of  $x_i$  squared. This drives each of the terms in the cost equation to one when their value approaches the maximum allowed, and to zero when their value is small.

## 3. Non-zero Steady State Outputs

The solution to the linear quadratic optimal feedback gain problem has been outlined in the previous sections. This solution assumes, however, that the desired steady-state value of the controlled states is zero. Since the Mark 6 control law includes nulling the sine of the inner or middle angle, it is possible that the desired value of the inner or middle angle be 180 degrees, not 0. In this case, the optimal feedback gain control law becomes:

$$u(t) = -F[-R+x(t)]$$

The vector  $R$  here represents the reference signal in the system, as shown in Figure 5.1. When  $R=0$ , the optimal control solution reduces to that shown above.

In addition to an axis shift, driving the system to a non-zero set point can also be accomplished by adding a constant factor  $u_0$  to the control input  $u(t)$ . The control input then becomes:

$$u(t) = -Fx(t) + u_0$$

A  $u_0$  can be chosen such that when the system experiences a shift in the set point, as the gyro controller does when shifting from 0 to 180 degrees, an optimal response is obtained while remaining within allowable input amplitudes. [Ref. 4: pp.270-275].

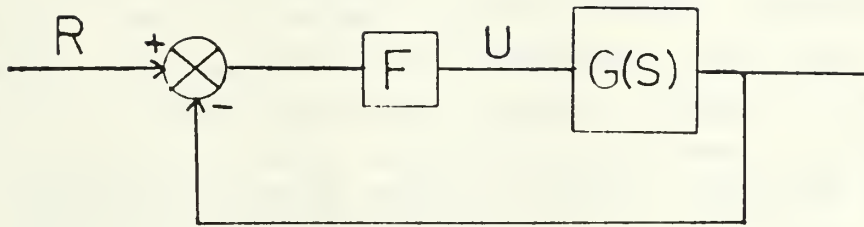


Figure 5.1 Optimal Gain Feedback Block Diagram.

This optimal constant input is defined as:

$$U_o = H_c^{-1}(0)z_o$$

In this equation,  $H_c(s)$  is the closed loop transfer function from the inputs to the desired non-zero steady state output  $z_o$ . The use of this solution requires that the number of inputs is equal to the number of outputs, and that the inverse of transfer function  $H_c(s)$  exists. If the transfer function has any zeros at the origin, the matrix  $H_c(0)$  is singular and the inverse does not exist.

The optimal solution for the gyro controller generates one or more zeros at the origin depending on the particular gimbal angles. Thus, the optimal input  $u_o$  cannot be found and the axis shift technique must be used.

## B. MARK 6 GYRO OPTIMAL CONTROLLER

### 1. Region Controller

With a sampling rate of 300 Hz, computing the optimal feedback gains every sampling interval would require an enormous amount of computations. Lauro has shown that a region controller will yield a nearly optimal response while greatly reducing the calculations involved [Ref. 1: p. 8].

The plant and control matrices of the Mark 6 gyro are a function of the gimbal angles, which in turn vary with time. The optimal regulator designed for the system divides the gimbal angle regions up into segments of 30 degrees or less. Optimal gains for the gimbal angles at the center of the region have been calculated. When the missile motion causes the gimbal angles to change from one region to another, the controller inserts the feedback gains calculated for that region. Figure 5.2 shows typical regional divisions for two angles. The regions are 10 degrees wide near zero, and 30 degrees wide further from the origin. The figure shows a two dimensional plot, although the actual gyro controller involves four angles in four dimensional space.

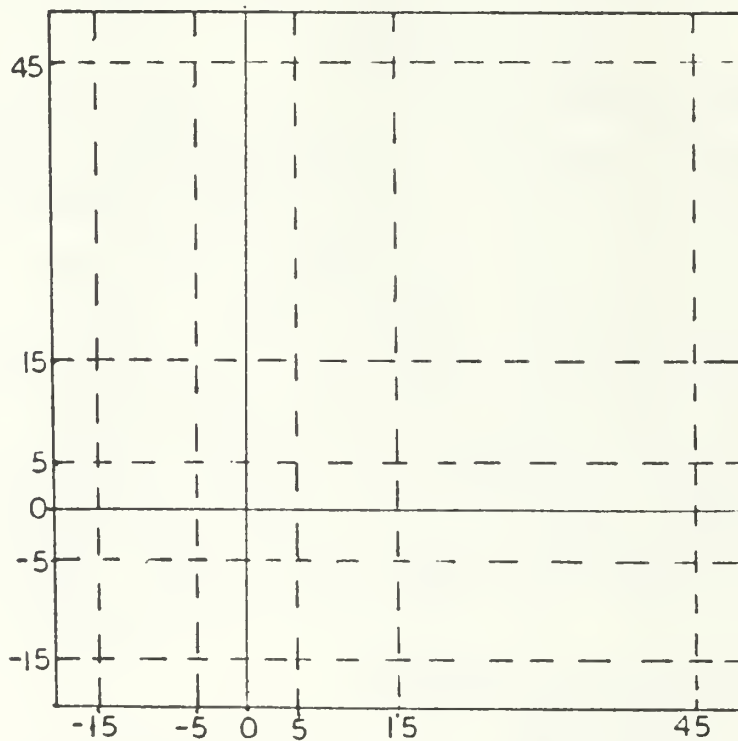


Figure 5.2 Region Controller Segments.

Figure 5.3 contains the algorithm of the computer program written to implement the optimal regulator. A copy of the program listing is included in Appendix C.

initialize missile motion and gyro rates to zero  
assign initial gimbal angles  
determine initial condition vector  $x(0)$   
    restrict all angles to  $\pm 360$  degrees  
    determine optimal gain region based on gimbal angles  
    assign appropriate optimal feedback gains  
    calculate variable terms in plant and control matrices  
    input missile motion  
    calculate  $x(k+1)$  values for each state

Figure 5.3    Algorithm for the Mark 6 Gyro Optimal Regulator.

## VI. SYSTEM SIMULATION

### A. SIMULATION MODEL

The model used for system simulation is a modified version of the state-space model shown in Figure 4.1. The U and V gyro errors are integrated, and the result nulled to minimize net gyro errors. The outer gimbal angle is calculated since it affects the plant and control matrices, but is not used for optimal feedback. The simulation model has eleven states, and is shown in Figure 6.1. The plant and control matrices for the simulation model are given in Figures 6.2 and 6.3.

### B. WEIGHTING MATRICES

#### 1. Torque Limits

Each axis of the gyro is driven by a torque motor. The amount of torque delivered by the motor is directly proportional to the feedback error signal. When the gyro errors are zero and the sine of the selected gimbal angle has been driven to null, no torque is generated.

The maximum torques which can be supplied to each axis are

- maximum inner axis torque  $T_{I\max}$  83 oz-in
- maximum middle axis torque  $T_{M\max}$  211 oz-in
- maximum outer axis torque  $T_{O\max}$  369 oz-in

Up to 14 percent more torque is available dependent upon the particular gimbal angles. This additional torque was not assumed to be available when designing the state-space system.

#### 2. State Output Weighting Matrix $R_1$

The optimal control cost function is defined as

$$J = \int [z^T(t)R_1(t)z(t) + u^T(t)R_2(t)u(t)]dt$$

The output measurement vector  $z(t)$  is

$$z(t) = [x_2(t) \ x_3(t) \ x_5(t) \ x_6(t) \ x_{10}(t) \ x_{11}(t)]^T$$



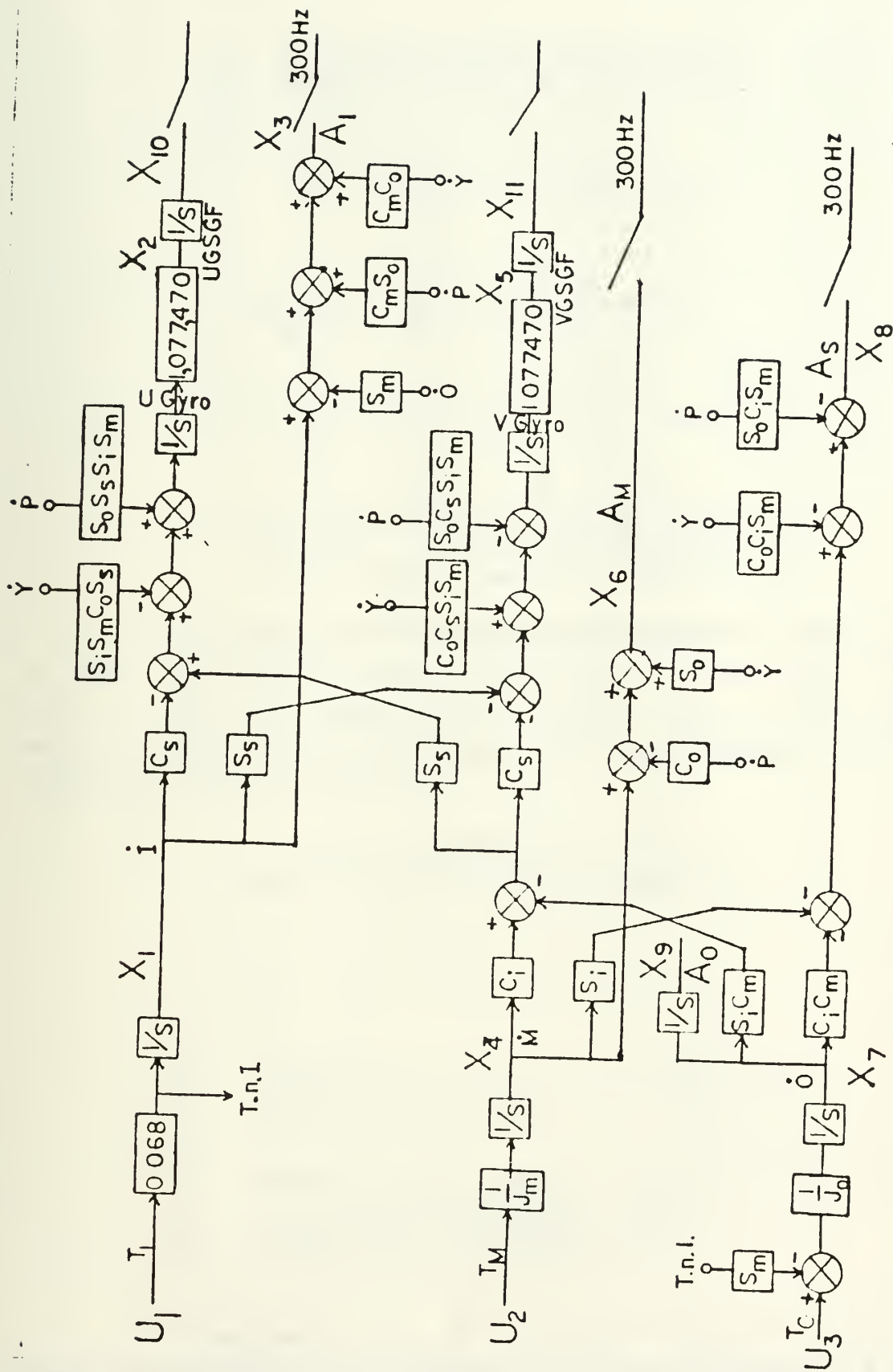


Figure 6.1 State-Space Simulation Model of the Mark 6 Gyro.



$$\begin{bmatrix} 0.068 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 1/J_M & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ -S_M/J_0 & 0 & 1/J_0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

Figure 6.3 Control Matrix for the  
State Space Simulation Model.

The output vector weighting matrix was chosen based on data provided by Draper Laboratory. The output vector varies with time depending on gimbal angles. The states which comprise the output vector and their maximum allowable values are shown in Table II.

TABLE II  
OUTPUT STATE VECTOR MAXIMUM VALUES

$x_2$	U Gyro	300 quanta
$x_3$	inner gimbal angle	30 degrees
$x_5$	V Gyro	300 quanta
$x_6$	middle gimbal angle	30 degrees
$x_{10}$	V Gyro error integral	1
$x_{11}$	U Gyro error integral	1

The maximum value of 300 quanta listed for the gyro angles corresponds to an error of 0.278 milliradians. This

is the maximum error attained with the current Mark 6 gimbal model. It was desired to allow a smaller error in the state-space model. The gyro has a physical stop at 2048 quanta, or 1.9 milliradians. If the error reaches this value the system will fail.

The inner and middle gimbal angle maximum values were chosen as 30 degrees. This prevents the product  $\text{SIN}(A_M)\text{SIN}(A_I)$  from exceeding 0.5.

The maximum value of the integrals of the U and V gyro errors was chosen arbitrarily as 1. This limit minimized gyro error without producing excessive input amplitudes.

The  $R_1$  matrix in terms of the output variables is

$$R_1 = \begin{bmatrix} 1/(x_2)_o^2 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1/(x_3)_o^2 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1/(x_5)_o^2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1/(x_6)_o^2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1/(x_{10})_o^2 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1/(x_{11})_o^2 \end{bmatrix}$$

Using the maximum values of the output states as given in Table II, The output cost matrix becomes

$$R_1 = \begin{bmatrix} 1.11 \times 10^{-5} & 0 & 0 & 0 & 0 & 0 \\ 0 & 3.65 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1.11 \times 10^{-5} & 0 & 0 & 0 \\ 0 & 0 & 0 & 3.65 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

### 3. Control Input Weighting Matrix $R_2$

The control input weighting matrix is a third order symmetric matrix with the inverse of the square of the allowable torques, given previously, on the diagonal. The matrix is shown below. The inputs are related to the torques as follows:

- $u_1$  = inner axis torque  $T_I$
- $u_2$  = middle axis torque  $T_M$
- $u_3$  = outer axis torque  $T_O$

$$R_2 = \begin{bmatrix} 1/(T_I)^2 & 0 & 0 \\ 0 & 1/(T_M)^2 & 0 \\ 0 & 0 & 1/(T_O)^2 \end{bmatrix}$$

Using the maximum torque values given previously, the  $R_2$  matrix is:

$$R_2 = \begin{bmatrix} 1.45 \times 10^{-4} & 0 & 0 \\ 0 & 2.25 \times 10^{-5} & 0 \\ 0 & 0 & 7.34 \times 10^{-6} \end{bmatrix}$$

## C. TEST PARAMETERS

### 1. Initial Conditions

The initial conditions used for the model simulation were provided by Draper Laboratory. This particular combination of initial conditions, along with the missile motion, has been determined to be the "worst case" situation for gyro control.

The gimbal angles have been initialized as follows:

- stable member gimbal angle - 0 degrees
- inner gimbal angle - 90 degrees
- middle gimbal angle - 0 degrees
- outer gimbal angle - 59.25 degrees

The system is started from rest, with all gimbal and angular rates zero. The initial condition vector  $z(0)$  is:

$$\begin{aligned} x_1(0) &= 0 \\ x_2(0) &= 0 \\ x_3(0) &= 90 \text{ degrees} \\ x_4(0) &= 0 \\ x_5(0) &= 0 \\ x_6(0) &= 0 \\ x_7(0) &= 0 \end{aligned}$$

$$\begin{aligned}x_8(0) &= 0 \\x_9(0) &= 59.25 \text{ degrees} \\x_{10}(0) &= 0 \\x_{11}(0) &= 0\end{aligned}$$

## 2. Missile Motion

The missile motion used for the simulation is a pitch rate input. The input is applied at  $8 \text{ rad/sec}^2$  to a steady-state value of  $1.571 \text{ rad/sec}$ , or  $90 \text{ deg/sec}$ . The input reaches steady-state after 0.19 seconds.

### D. TEST RESULTS

The system was initialized using the values above, and the pitch input applied. The tabulated data for the simulation run is included in Appendix D.

Figure 6.4 shows a plot of U and V gyro error as a function of time. The switching points between optimal gain regions are shown on the figure. The error graphs show a transient error spike when the optimal gains are changed. The amplitude of this spike can be made arbitrarily small by reducing the size of the optimal control regions. If optimal gains were calculated and applied every sample interval, the curve would show no such fluctuations.

A comparison between the errors of the Draper lab Mark 6 model and the state-space model with optimal control region gains is shown in Figures 6.5 and 6.6. The UGSGF error signal, Figure 6.5, closely approximates the Draper model with the exception of the switching point transients. The error damps rapidly after the optimal gain change, maintaining a small steady state error. The pitch input to the U gyro has a gain including both the  $\text{SIN}(A_M) * \text{SIN}(A_I)$  term and a  $\text{SIN}(A_S)$  term, as seen in Figure 4.1. Since both the middle and stable gimbal angles are initially zero, the pitch rate does not affect the U gyro significantly at the start of the simulation run.



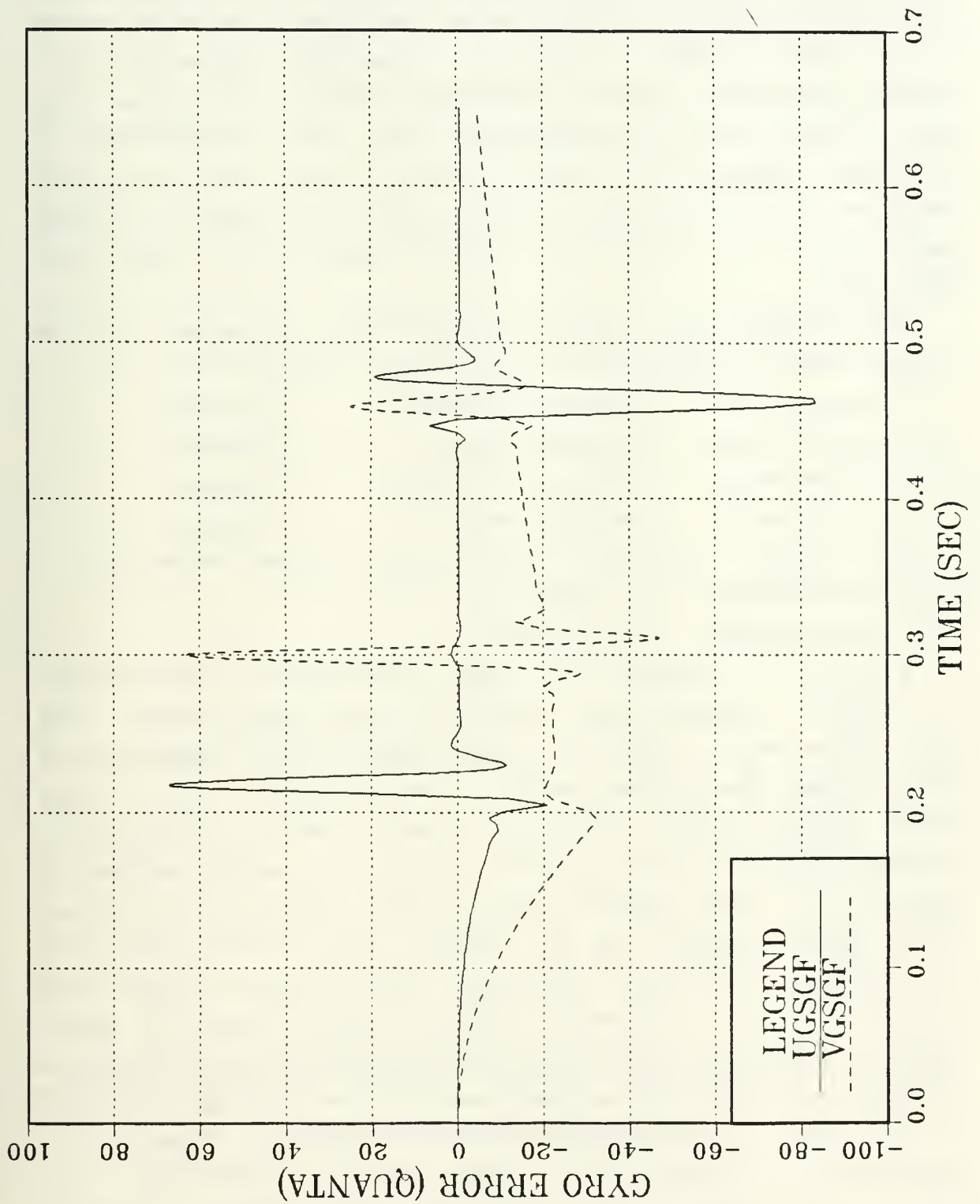


Figure 6.4 U and V gyro Errors for the State-Space Model.

The VGSGF error signal graph, Figure 6.6, shows a less pronounced response to missile motion than that experienced by the Draper model. The error transients again damp out quickly, with the system approaching but not attaining zero error at the end of the simulation run. The V gyro shows an immediate response to missile motion since the gain block contains a  $\cos(A_S)$  term, allowing the pitch rate to affect the gyro error as soon as the middle gimbal angle moves away from zero.

The maximum gyro errors of the Draper lab model and the optimal model, neglecting the transient error spikes, are

- U gyro error      Draper lab model      12 quanta
- U gyro error      optimal model      10 quanta
- V gyro error      Draper lab model      94 quanta
- V gyro error      optimal model      31 quanta

The maximum transient errors of the optimal model are

- U gyro error      83 quanta
- V gyro error      63 quanta

Figure 6.7 compares the inner versus middle gimbal angle of the Draper model and the state-space model. The state-space model exhibits a more rapid middle gimbal angle change than the Draper model. The angular rate of the middle gimbal angle could be slowed down by varying the maximum allowed value of the angle in the  $R_2$  weighting matrix. A slower angular rate would result in a smaller middle gimbal angle, and the  $\sin(A_M)$  term in the gain path would reduce the effect the missile motion had on the U and V gyro errors. A trade-off between the two must be made, however, since the input signals required to hold the middle angle at a slower rate may adversely affect the gyro errors.

The inner gimbal angles of both systems translate at approximately the same rate. This can be expected since the sine of the middle angle, not the inner angle, is being nulled at the start of the simulation. The inner gimbal angle is not being controlled.

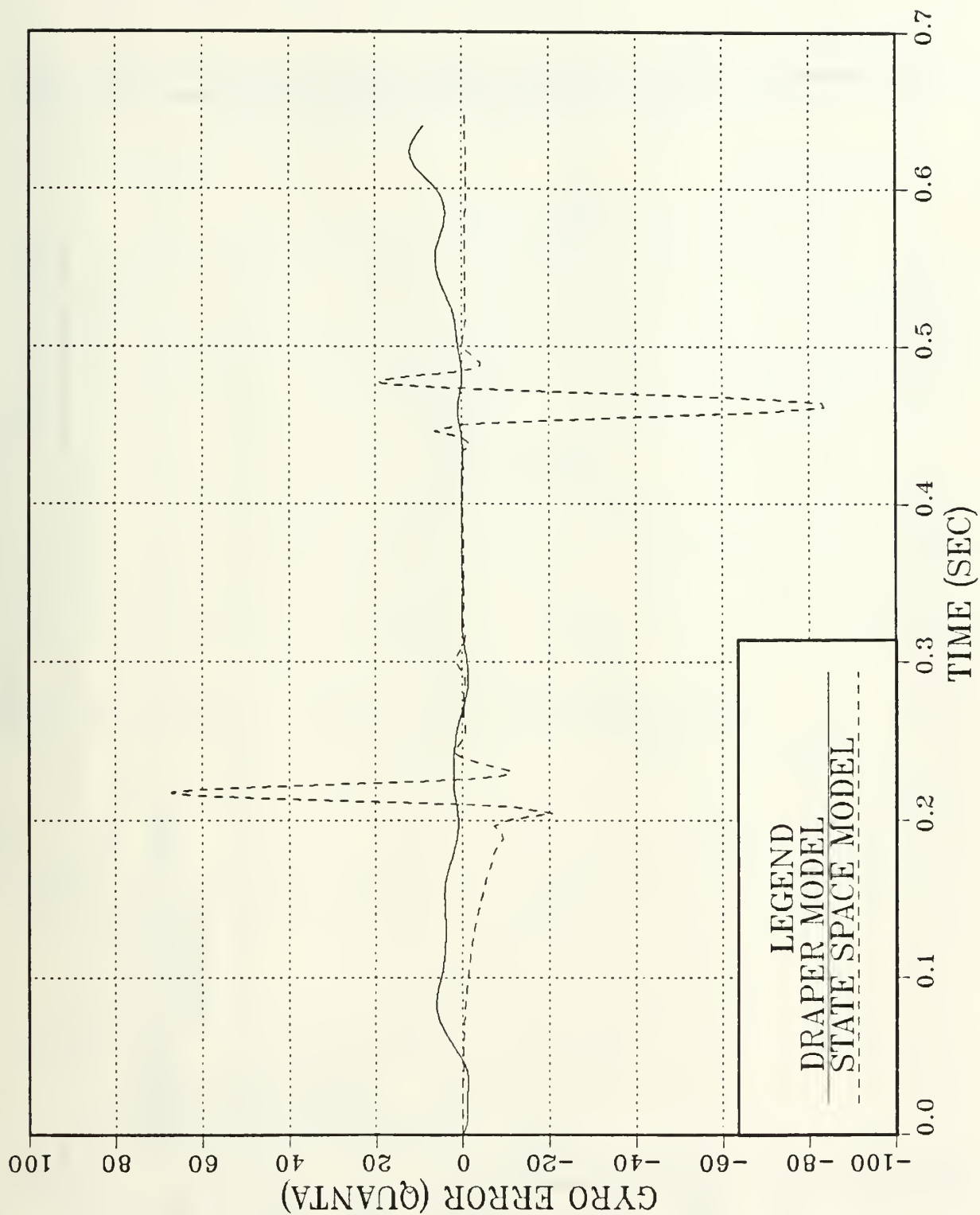


Figure 6.5 UGSGF Errors - Draper Lab and State-Space Models.

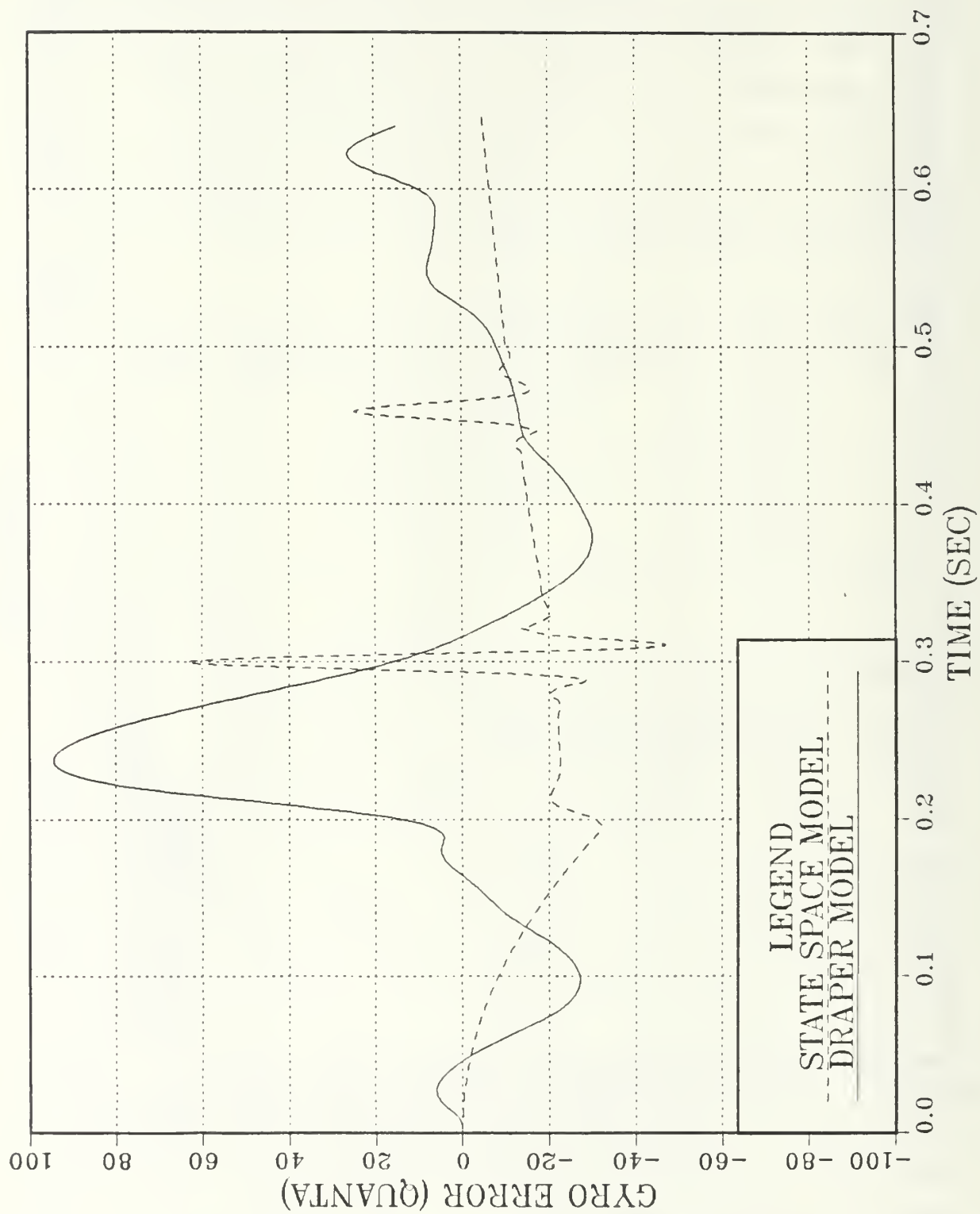


Figure 6.6 VGSGF Errors - Draper Lab and State-space models.

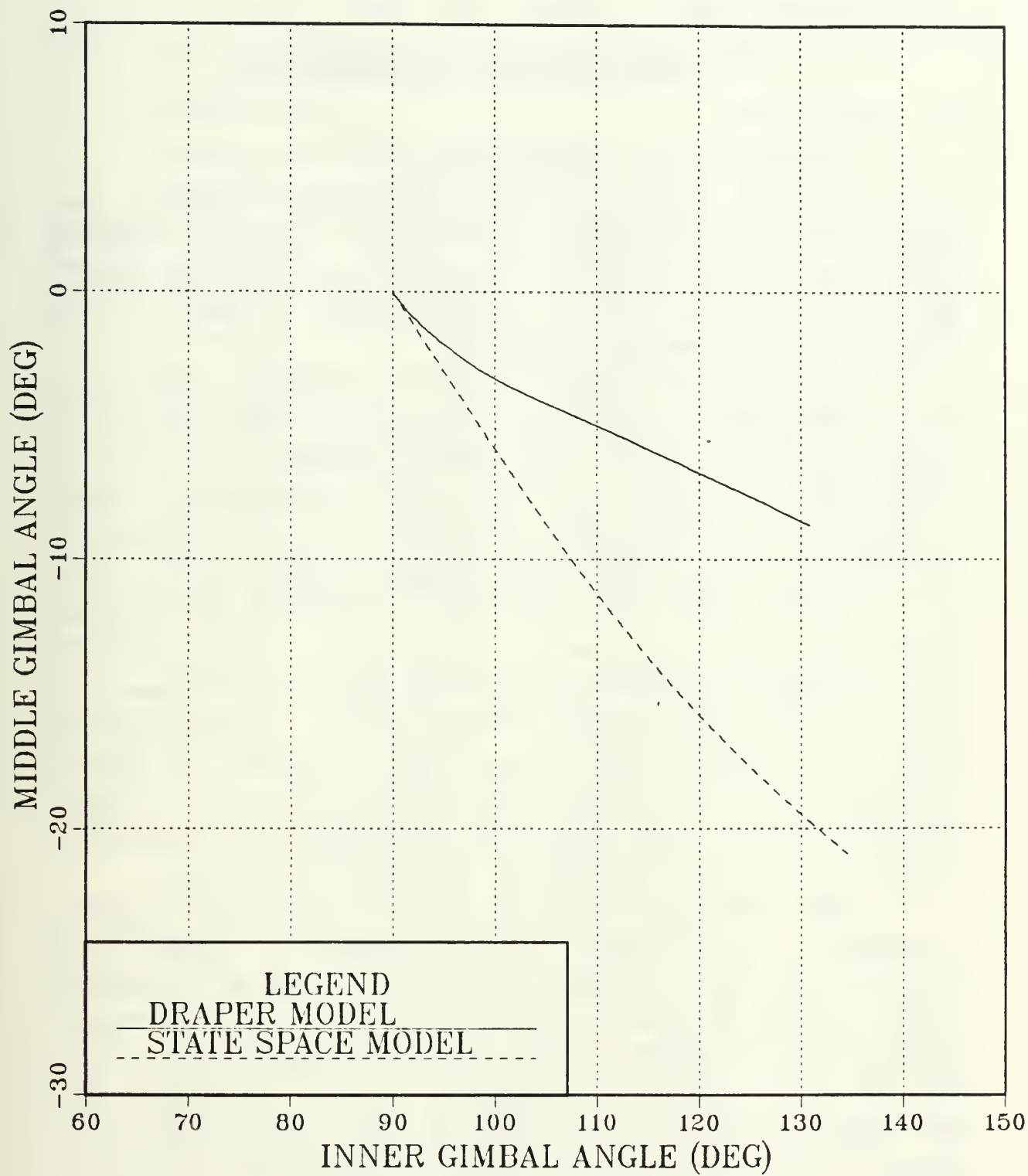


Figure 6.7 Inner and Middle Gimbal Angles.

## VII. CONCLUSIONS AND RECOMMENDATIONS

### A. CONCLUSIONS

#### 1. Optimal Design Techniques

It has been shown that an optimal feedback gain controller can successfully control gyro errors and prevent gimbal lock. The unique, non-linear nature of the Mark 6 Gyro requires that several modifications be made to the linear quadratic problem solution.

As the gimbal angles vary, the internal plant gains vary. Depending on the gyro orientation, these gains can range from zero to one. Care must be taken in the regions surrounding zero gain, since the loop is effectively broken at this point. If the optimal control regions chosen are too large, the instantaneous switching of the gain from zero to a finite value may cause large transient errors from which the system is unable to recover. Narrowing the optimal control regions near the zero points will lessen the impact of the loop suddenly being closed. Increasing the number of control regions would also increase the memory size required to store the additional gain values. There exists a trade-off between system performance and the amount of control hardware required to implement the system.

The constantly changing gimbal angles also lead to numerous set point shifts. It was shown in chapter five that there exists an optimal control input which drives the state to the desired value without exceeding the input amplitude limitations. This solution requires that the inverse of the closed loop transfer function exists and that the number of controlled states is equal to the number of inputs. [Ref. 4: p. 279]. If either of these conditions is not met, a set point axis shift must be used, which may cause large transient errors. Narrowing the optimal control



regions in these areas may improve system performance. Again, additional memory space would be required.

## B. RECOMMENDATIONS

### 1. Improving System Performance

The gyro controller used in the simulation was designed using the linear quadratic optimal feedback gain design technique. Since the technique provides optimal control only at the design point, a region controller will not provide optimal response to missile motion throughout the region. Changing the weighting matrices  $R_1$  and  $R_2$  for a particular region may provide improved system performance while still remaining within the allowable torque limits. Since the maximum allowable value for the controlled gimbal angle and gyro error integrals are essentially arbitrary, they can be varied and the resulting system performance analyzed to determine if the error in that region can be reduced.

The simulation model also neglected the effects of friction, which can reduce net torque to the axes by 3 to 4 oz-in. The friction effects, which are a function of gimbal angular rate, may produce a marked decrease in system performance in areas where the angles are changing rapidly.

Finally, the model assumed that all states were available for feedback and that there was no noise input. In practical applications, this assumption is invalid. A Kalman filter must be designed for the system to provide an estimate of the states which can not be measured, and to optimize system performance in the presence of both process and measurement noise. Harmonics from the gyros and the torque motors may have an effect on the system, along with structural noise generated during missile flight. The non-linearity and time varying nature of the system make the design of a filter particularly complex.

APPENDIX A  
DEFINITIONS

TABLE III  
VARIABLE DEFINITIONS

$A_S$	stable element gimbal angle
$A_I$	inner gimbal angle
$A_M$	middle gimbal angle
$A_O$	outer gimbal angle
$\dot{S}$	stable member inertial rate
$\dot{I}$	inner gimbal inertial rate
$\dot{M}$	middle gimbal inertial rate
$\dot{O}$	outer gimbal inertial rate
$S_S$	sine of stable element angle
$C_S$	cosine of stable element angle
$S_I$	sine of inner gimbal angle
$C_I$	cosine if inner gimbal angle
$S_M$	sine of middle gimbal angle
$C_M$	cosine of middle gimbal angle
$S_O$	sine of outer gimbal angle
$C_O$	cosine of outer gimbal angle
UGSGF	U gyro error signal expressed in quanta
VGSGF	V gyro error signal expressed in quanta
IMU	Inertial Measurement Unit the gyro and its controller

## APPENDIX B

### A

#### 1. FOUR GIMBAL IMU MODEL

This appendix develops the equations which govern the gyro motion due to inputs from missile motion and torque motors. A block diagram of the plant is developed from the equations, which also leads to the state-space model [Ref. 2: pp. 2-7].

##### 1. Assumptions and Approximations

This analysis includes the following assumptions and approximations:

- Spherical inertias
- Small torques due to gyroscopic effects

##### 2. Definitions

Gimbal motion is positive in a clockwise direction. The transformation from an inner member to an outer member is then in the form:

$$\begin{array}{ccc} \text{inner member} & & \\ C & = & \begin{bmatrix} \text{COS} & -\text{SIN} & 0 \\ \text{SIN} & \text{COS} & 0 \\ \text{COS} & 0 & 1 \end{bmatrix} \\ \text{outer member} & & \end{array}$$

- Standard i,j,k axes will be used.
- x, y and z are rate components on the i, j and k axes.
- p, q and r are body, or missile, motion rates on the i, j and k axes representing pitch, roll and yaw respectively.
- A positive torque T applied to a gimbal causes a positive, clockwise rotation.
- Ex, Ey and Ez are the gyro errors sensed on the stable, or inertial, member.

### 3. Symbols

- $\dot{S}$  - Stable member gimbal rate resulting from applied torque
- $\dot{I}$  - Inner member gimbal rate resulting from applied torque
- $\dot{M}$  - Middle member gimbal rate resulting from applied torque
- $\dot{O}$  - Outer member gimbal rate resulting from applied torque
- $\dot{A}_s$  - Stable member gimbal rate resulting from relative motion between gimbals
- $\dot{A}_i$  - Inner member gimbal rate resulting from relative motion between gimbals
- $\dot{A}_m$  - Middle member gimbal rate resulting from relative motion between gimbals
- $\dot{A}_o$  - Outer member gimbal rate resulting from relative motion between gimbals

### 4. Transformation of missile motion to inertial member

#### a. Base motion

Base motion  $W_b(5) = p_{i5} + q_{j5} + r_{k5}$

referred to outer gimbal: (degree of freedom along  $j_4$  and  $j_5$ )

$$\left. W_b \right|_0 = \begin{bmatrix} C_o & 0 & -S_o \\ 0 & 1 & 0 \\ S_o & 0 & C_o \end{bmatrix} \begin{matrix} p \\ q \\ r \end{matrix}$$

$$\left. \begin{matrix} W_{bx} \\ W_{by} \\ W_{bz} \end{matrix} \right|_{O(4)} = \begin{bmatrix} C_o * p - S_o * r \\ q \\ S_o * p + C_o * r \end{bmatrix}$$

Rates along  $W_{by}$  do not couple to outer gimbal.  $O$  is a rate applied to the outer gimbal from its torque motor.

$$\begin{array}{c|c} \begin{array}{l} W_{ox} \\ W_{oy} \\ W_{oz} \end{array} & = \begin{bmatrix} Co*p - So*r \\ 0 \\ So*p + Co*r \end{bmatrix} \\ \hline & 0(4) \end{array}$$

b. Outer gimbal to middle gimbal transformation

Referred to middle gimbal: (degree of freedom along the i3 and i4 axes)

$$\begin{array}{c|c} \begin{array}{l} W_{ox} \\ W_{oy} \\ W_{oz} \end{array} & = \begin{bmatrix} 1 & 0 & 0 \\ 0 & C_m & -S_m \\ 0 & S_m & C_m \end{bmatrix} \begin{bmatrix} Co*p - So*r \\ 0 \\ So*p + Co*r \end{bmatrix} \\ \hline & M(3) \end{array}$$

$$= \begin{bmatrix} Co*p - So*r \\ C_m*O - S_m(So*p + Co*r) \\ S_m*O + C_m(So*p + Co*r) \end{bmatrix}$$

Rates along  $W_{ox|m}$  do not couple to middle gimbal. M is the rate of middle gimbal applied by its torque motor.

$$\begin{array}{c|c} \begin{array}{l} W_{mk} \\ W_{my} \\ W_{mz} \end{array} & = \begin{bmatrix} M \\ C_m*O - S_m*(So*p + Co*p) \\ S_m*O + C_m*(So*p + Co*r) \end{bmatrix} \\ \hline & M(3) \end{array}$$

c. Middle gimbal to inner gimbal transformation.

Referred to inner gimbal (degree of freedom along k2 and k3 axes):

$$\begin{matrix} W_{mx} \\ W_{my} \\ W_{mz} \end{matrix} \bigg|_{I(2)} = \begin{bmatrix} C_i & -S_i & 0 \\ S_i & C_i & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} M \\ C_m*O - S_m*(S_o*p + C_o*r) \\ S_m*O + C_m*(S_o*p + C_o*r) \end{bmatrix}$$

$$= \begin{bmatrix} C_i*M - S_i*EC_m*O - S_m*(S_o*p + C_o*r)' \\ S_i*M + C_i*EC_m*O - S_m*(S_o*p + C_o*r)T \\ S_m*O + C_m*(S_o*p + C_o*r) \end{bmatrix}$$

Rates along  $W_{mz}|I$  do not couple into inner gimbal.  $I$  is rate on inner gimbal applied by torque motor, therefore:

$$\begin{matrix} W_{ix} \\ W_{iy} \\ W_{iz} \end{matrix} \bigg|_{I(2)} = \begin{bmatrix} C_i*M - S_i*EC_m*O - S_m*(S_o*p + C_o*r)T \\ S_i*M + C_i*EC_m*O - S_m*(S_o*p + C_o*r)T \\ I \end{bmatrix}$$

d. Inner gimbal to stable member transformation.

Referred to stable member (degree of freedom along  $j_1$  and  $j_2$  axes):

$$\begin{matrix} W_{ix} \\ W_{iy} \\ W_{iz} \end{matrix} \bigg|_{S(1)} = \begin{bmatrix} C_s & 0 & -S_s \\ 0 & 1 & 0 \\ S_s & 0 & C_s \end{bmatrix} \begin{bmatrix} W_{ix} \\ W_{iy} \\ W_{iz} \end{bmatrix}$$

$$= \begin{bmatrix} C_s*W_{ix} - S_s*W_{iz} \\ W_{iy} \\ S_s*W_{ix} + C_s*W_{iz} \end{bmatrix}$$

Rates along  $W_{iy}|S$  do not couple across.  $E$  is rate on stable member applied by motor, therefore:



$$\begin{array}{c|c} \text{Wex} \\ \text{Wey} \\ \text{Wez} \\ \hline \text{S} \end{array} = \begin{bmatrix} \text{Cs}^*[\text{Ci}^*\text{M} - \text{Si}^*[\text{Cm}^*\text{O} - \text{Sm}^*(\text{So}^*\text{p} + \text{Co}^*\text{r})]] - \text{Ss}^*\text{I} \\ \text{E} \\ \text{Ss}^*[\text{Ci}^*\text{M} - \text{Si}^*[\text{Cm}^*\text{O} - \text{Sm}^*(\text{So}^*\text{p} + \text{Co}^*\text{r})]] + \text{Cs}^*\text{I} \end{bmatrix}$$

e. Four Gimbal Block Diagram.

The following expressions are used to develop the four gimbal IMU block diagram shown in Figure B.1:

$$\dot{\text{A}}_{\text{E}} = \text{W}_{\text{EY}}\text{j}_1 - \text{W}_{\text{IY}}\text{j}_2$$

$$\dot{\text{A}}_{\text{I}} = \text{W}_{\text{IZ}}\text{k}_2 - \text{W}_{\text{MZ}}\text{k}_3$$

$$\dot{\text{A}}_{\text{M}} = \text{W}_{\text{MX}}\text{i}_3 - \text{W}_{\text{OX}}\text{i}_4$$

$$\dot{\text{A}}_{\text{O}} = \text{W}_{\text{OY}}\text{j}_4 - \text{W}_{\text{BY}}\text{j}_5$$

$$\text{E}_{\text{Y}} = \text{Ej}_1$$

$$\text{E}_{\text{X}} = \text{W}_{\text{EX}}\text{i}_1$$

$$\text{E}_{\text{Z}} = \text{W}_{\text{EZ}}\text{k}_1$$

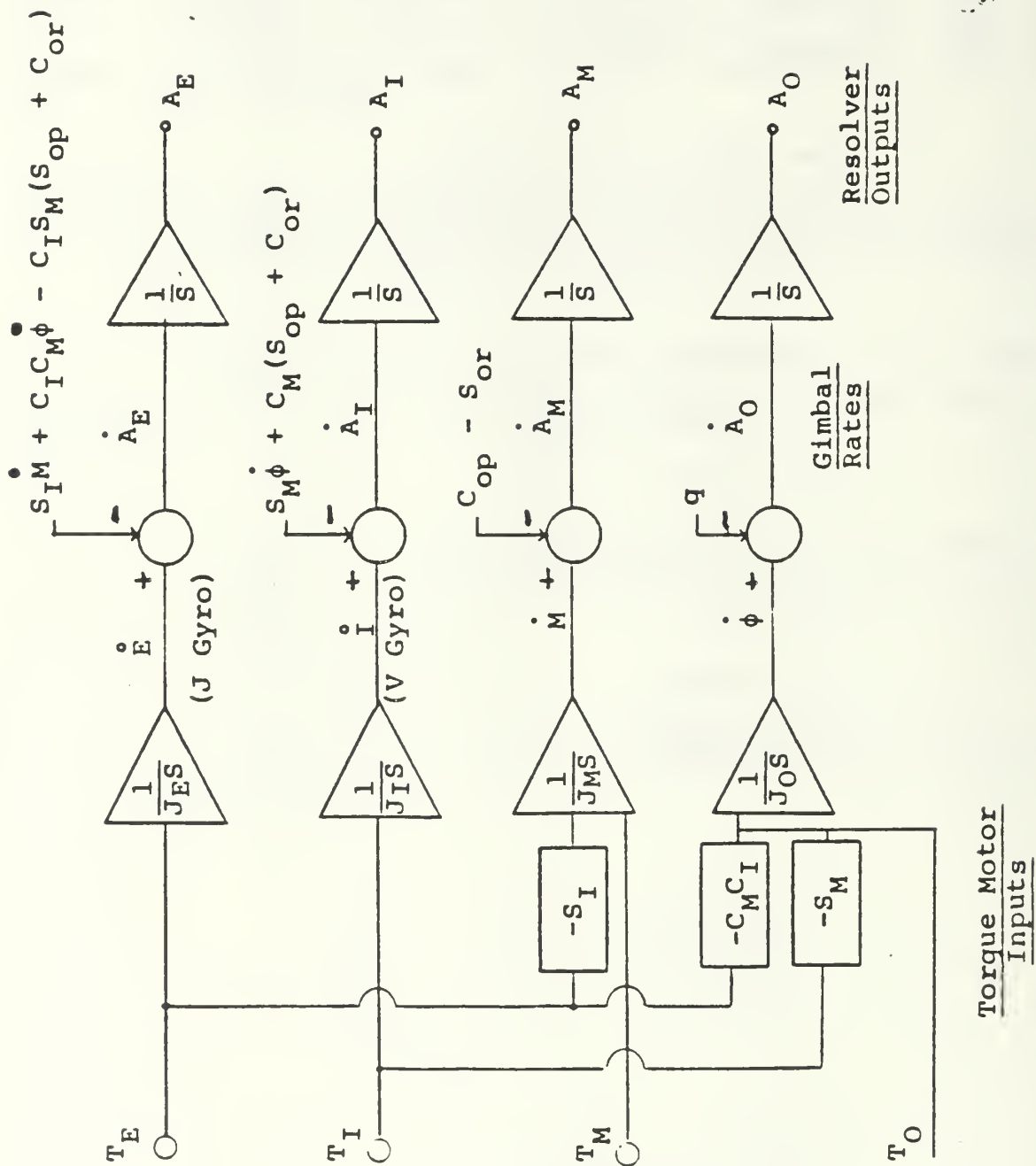


Figure B.1 Four Gimbal Gyro Block Diagram.

## A1

UUUUUUUUUUUUUUUUUUUUUU    U    UUUUUUUU                 U                 UUU          UU

```

C          ASSIGN INITIAL CONDITION VECTOR
X1{1}=0
X2{1}=0
X3{1}=ANGI
X4{1}=0
X5{1}=0
X6{1}=ANGM
X7{1}=0
X8{1}=ANGS
X9{1}=ANGO
X10{1}=0
X11{1}=0

CCC      P=0
          T=0.000333333
          TWOPI=6.283185308
          PI=3.14159

CCCC     STEP THROUGH ROUTINE 2800 TIMES

CCCCC    DO 10 K=1,2800

          RESTRICT GIMBAL ANGLES TO BETWEEN -360 AND +360

          IF (X3(K).GE.TWOPI) THEN
            X3(K)=X3(K)-TWOPI
            IF (X3(K).GE.TWOPI) GO TO 1
          ELSE IF (X3(K).LE.-TWOPI) THEN
            X3(K)=X3(K)+TWOPI
            IF (X3(K).LE.-TWOPI) GO TO 1
          END IF
          IF (X6(K).GE.TWOPI) THEN
            X6(K)=X6(K)-TWOPI
            IF (X6(K).GE.TWOPI) GO TO 2
          ELSE IF (X6(K).LE.-TWOPI) THEN
            X6(K)=X6(K)+TWOPI
            IF (X6(K).LE.-TWOPI) GO TO 2
          END IF
          IF (X8(K).GE.TWOPI) THEN
            X8(K)=X8(K)-TWOPI
            IF (X8(K).GE.TWOPI) GO TO 3
          ELSE IF (X8(K).LE.-TWOPI) THEN
            X8(K)=X8(K)+TWOPI

```

```

OPT00490
OPT00500
OPT00510
OPT00520
OPT00530
OPT00540
OPT00550
OPT00560
OPT00570
OPT00580
OPT00590
OPT00600
OPT00610
OPT00620
OPT00630
OPT00640
OPT00650
OPT00660
OPT00670
OPT00680
OPT00690
OPT00700
OPT00710
OPT00720
OPT00730
OPT00740
OPT00750
OPT00760
OPT00770
OPT00780
OPT00790
OPT00800
OPT00810
OPT00820
OPT00830
OPT00840
OPT00850
OPT00860
OPT00870
OPT00880
OPT00890
OPT00900
OPT00910
OPT00920
OPT00930
OPT00940
OPT00950
OPT00960

```





```

C1100 IF (ANGM.LT.-45) GO TO 999 OPT01450
      IF {ANGM.GE.-45 .AND. ANGM.LT.-22.5} GO TO 1140 OPT01460
      IF {ANGM.GE.-22.5 .AND. ANGM.LT.-15} GO TO 1130 OPT01470
      IF {ANGM.GE.-15 .AND. ANGM.LT.-5} GO TO 1120 OPT01480
      IF {ANGM.GE.-5 .AND. ANGM.LE.5} GO TO 1110 OPT01490
      IF {ANGM.GT.5} GO TO 999 OPT01500
      OPT01510
      OPT01520
      OPT01530
      OPT01540
      OPT01550
      OPT01560
      OPT01570
      OPT01580
      OPT01590
      OPT01600
      OPT01610
      OPT01620
      OPT01630
      OPT01640
      OPT01650
      OPT01660
      OPT01670
      OPT01680
      OPT01690
      OPT01700
      OPT01710
      OPT01720
      OPT01730
      OPT01740
      OPT01750
      OPT01760
      OPT01770
      OPT01780
      OPT01790
      OPT01800
      OPT01810
      OPT01820
      OPT01830
      OPT01840
      OPT01850
      OPT01860
      OPT01870
      OPT01880
      OPT01890
      OPT01900
      OPT01910
      OPT01920

C*****
C1110 IF (ANGI.LT.75) GO TO 999
      IF {ANGI.GE.75 .AND. ANGI.LE.105} GO TO 1111
      IF {ANGI.GE.105 .AND. ANGI.LT.135} GO TO 1112
      IF {ANGI.GT.135} GO TO 999
      OPT01590
      OPT01600
      OPT01610
      OPT01620
      OPT01630
      OPT01640
      OPT01650
      OPT01660
      OPT01670
      OPT01680
      OPT01690
      OPT01700
      OPT01710
      OPT01720
      OPT01730
      OPT01740
      OPT01750
      OPT01760
      OPT01770
      OPT01780
      OPT01790
      OPT01800
      OPT01810
      OPT01820
      OPT01830
      OPT01840
      OPT01850
      OPT01860
      OPT01870
      OPT01880
      OPT01890
      OPT01900
      OPT01910
      OPT01920

C*****
C1120 IF (ANGI.LT.75) GO TO 999
      IF {ANGI.GE.75 .AND. ANGI.LT.105} GO TO 1122
      IF {ANGI.GE.105 .AND. ANGI.LT.135} GO TO 1121
      IF {ANGI.GE.135} GO TO 999
      OPT01590
      OPT01600
      OPT01610
      OPT01620
      OPT01630
      OPT01640
      OPT01650
      OPT01660
      OPT01670
      OPT01680
      OPT01690
      OPT01700
      OPT01710
      OPT01720
      OPT01730
      OPT01740
      OPT01750
      OPT01760
      OPT01770
      OPT01780
      OPT01790
      OPT01800
      OPT01810
      OPT01820
      OPT01830
      OPT01840
      OPT01850
      OPT01860
      OPT01870
      OPT01880
      OPT01890
      OPT01900
      OPT01910
      OPT01920

C*****
C1130 IF (ANGI.LT.105) GO TO 999
      IF {ANGI.GE.105 .AND. ANGI.LT.135} GO TO 1131
      IF {ANGI.GE.135 .AND. ANGI.LT.165} GO TO 1132
      IF {ANGI.GE.165} GO TO 999
      OPT01590
      OPT01600
      OPT01610
      OPT01620
      OPT01630
      OPT01640
      OPT01650
      OPT01660
      OPT01670
      OPT01680
      OPT01690
      OPT01700
      OPT01710
      OPT01720
      OPT01730
      OPT01740
      OPT01750
      OPT01760
      OPT01770
      OPT01780
      OPT01790
      OPT01800
      OPT01810
      OPT01820
      OPT01830
      OPT01840
      OPT01850
      OPT01860
      OPT01870
      OPT01880
      OPT01890
      OPT01900
      OPT01910
      OPT01920

C*****
C1140 IF (ANGI.LT.135) GO TO 999
      IF {ANGI.GE.135 .AND. ANGI.LT.165} GO TO 1141
      IF {ANGI.GE.165} GO TO 999
      OPT01590
      OPT01600
      OPT01610
      OPT01620
      OPT01630
      OPT01640
      OPT01650
      OPT01660
      OPT01670
      OPT01680
      OPT01690
      OPT01700
      OPT01710
      OPT01720
      OPT01730
      OPT01740
      OPT01750
      OPT01760
      OPT01770
      OPT01780
      OPT01790
      OPT01800
      OPT01810
      OPT01820
      OPT01830
      OPT01840
      OPT01850
      OPT01860
      OPT01870
      OPT01880
      OPT01890
      OPT01900
      OPT01910
      OPT01920

C*****
C1200 IF (ANGM.LT.-45) GO TO 999
      IF {ANGM.GE.-45 .AND. ANGM.LT.-15} GO TO 1210
      IF {ANGM.GE.-15 .AND. ANGM.LT.-5} GO TO 1220
      IF {ANGM.GT.-5} GO TO 999
      OPT01590
      OPT01600
      OPT01610
      OPT01620
      OPT01630
      OPT01640
      OPT01650
      OPT01660
      OPT01670
      OPT01680
      OPT01690
      OPT01700
      OPT01710
      OPT01720
      OPT01730
      OPT01740
      OPT01750
      OPT01760
      OPT01770
      OPT01780
      OPT01790
      OPT01800
      OPT01810
      OPT01820
      OPT01830
      OPT01840
      OPT01850
      OPT01860
      OPT01870
      OPT01880
      OPT01890
      OPT01900
      OPT01910
      OPT01920

C*****
C1210 IF (ANGI.LT.135) GO TO 999
      IF {ANGI.GE.135 .AND. ANGI.LT.165} GO TO 1211
      IF {ANGI.GT.165} GO TO 999
      OPT01590
      OPT01600
      OPT01610
      OPT01620
      OPT01630
      OPT01640
      OPT01650
      OPT01660
      OPT01670
      OPT01680
      OPT01690
      OPT01700
      OPT01710
      OPT01720
      OPT01730
      OPT01740
      OPT01750
      OPT01760
      OPT01770
      OPT01780
      OPT01790
      OPT01800
      OPT01810
      OPT01820
      OPT01830
      OPT01840
      OPT01850
      OPT01860
      OPT01870
      OPT01880
      OPT01890
      OPT01900
      OPT01910
      OPT01920

```



```

C*****OPT01930
C*****OPT01940
1220 IF (ANGI.LT.135) GO TO 999OPT01950
      IF (ANGI.GE.135.AND. ANGI.LT.165) GO TO 1221OPT01960
      IF (ANGI.GT.165) GO TO 999OPT01970
C*****OPT01980
C*****OPT01990
C*****OPT02000
C*****OPT02010
C*****OPT02020
1111 OPTIMAL GAINS FOR ANGS=0 ANGI=90 ANGM=0 ANGO=60OPT02030
      F11=0.204226E+04OPT02040
      F12=-.131298E+00OPT02050
      F13=-.815389E-22OPT02060
      F14=0.182125E+01OPT02070
      F15=0.186264E-01OPT02080
      F16=0.00000E+00OPT02090
      F17=0.520824E+02OPT02100
      F18=-.783169E-40OPT02110
      F19=0.305267E+01OPT02120
      F21=-.327388E+01OPT02130
      F22=0.206725E-03OPT02140
      F23=-.845929E-15OPT02150
      F24=0.269129E+02OPT02160
      F25=0.226333E+00OPT02170
      F26=0.00000E+00OPT02180
      F27=0.769628E+03OPT02190
      F28=-.478709E-34OPT02200
      F29=0.355655E+02OPT02210
      F31=-.107285E+03OPT02220
      F32=0.677435E-02OPT02230
      F33=0.791307E-16OPT02240
      F34=0.881931E+03OPT02250
      F35=0.741688E+01OPT02260
      F36=0.00000E+00OPT02270
      F37=0.252206E+05OPT02280
      F38=0.447798E-35OPT02290
      F39=0.116548E+04OPT02300
      GO TO 50OPT02310
C*****ANGS=0 ANGI=120 ANGM=-5 ANGO=60OPT02320
C*****F11=0.204140E+04OPT02330
1112 F12=-.131289E+00OPT02340
      F13=0.184866E-08OPT02350
      F14=0.244617E+02OPT02360
      F15=0.539820E-01OPT02370
      F16=0.00000E+00OPT02380
      F17=0.112314E+03OPT02390
      F18=0.464785E-24OPT02400

```

```

E119=0. 134149E+02
E121=-. 533123E+02
E223=0. 303621E-027
E224=0. 134458E-07
E225=0. 517502E+04
E226=0. 487442E+01
E227=0. 000000E+00
E228=0. 943919E+04
E229=0. 178859E-23
E331=-. 118859E+04
E332=0. 127164E+03
E333=0. 778576E-027
E334=0. 124747E-07
E335=0. 136002E+05
E336=0. 124995E+02
E337=0. 000000E+00
E338=0. 239397E+05
E339=0. 536589E-23
GO TO 50

```

C  
1121

ANGS=0 ANGI=120 ANGM=-10 ANGO=60

```

OPT02410
OPT02420
OPT02430
OPT02440
OPT02450
OPT02460
OPT02470
OPT02480
OPT02490
OPT02500
OPT02510
OPT02520
OPT02530
OPT02540
OPT02550
OPT02560
OPT02570
OPT02580
OPT02590
OPT02600
OPT02610
OPT02620
OPT02630
OPT02640
OPT02650
OPT02660
OPT02670
OPT02680
OPT02690
OPT02700
OPT02710
OPT02720
OPT02730
OPT02740
OPT02750
OPT02760
OPT02770
OPT02780
OPT02790
OPT02800
OPT02810
OPT02820
OPT02830
OPT02840
OPT02850
OPT02860
OPT02870
OPT02880

```

F37=0. 160605E+05  
 F38=0. 913901E-32  
 F39=0. 962472E+03  
 F310=0. 203602E+02  
 CO TO 50  
 ANGS=0 ANGI=90 ANGM=-10 ANGO=60  
 F11=0. 790346E+04  
 F12=0. 196704E+01  
 F13=0. 301838E-07  
 F14=0. 142808E+03  
 F15=0. 291412E-01  
 F16=0. 17578E-24  
 F17=0. 705603E+02  
 F18=0. 200938E-24  
 F19=0. 634065E+01  
 F110=0. 262536E+03  
 F221=0. 563134E+02  
 F222=0. 119556E-01  
 F223=0. 987899E-08  
 F224=0. 130813E+02  
 F225=0. 340290E+00  
 F226=0. 470838E-26  
 F227=0. 114722E+04  
 F228=0. 148104E-25  
 F229=0. 534212E+02  
 F2210=0. 129020E+01  
 F231=0. 122608E+04  
 F232=0. 260306E+00  
 F233=0. 214697E-06  
 F234=0. 286723E+03  
 F235=0. 740904E+01  
 F236=0. 103164E-24  
 F237=0. 249778E+05  
 F238=0. 318815E-24  
 F239=0. 116312E+04  
 F310=0. 280912E+02  
 CO TO 50

C  
 C  
 1122

ANGS=-30 ANGI=150 ANGM=-30 ANGO=60  
 F11=0. 178657E+04  
 F12=0. 998658E-01  
 F13=0. 248111E-08  
 F14=0. 756883E+03  
 F15=0. 589541E+00  
 F16=0. 000000E+00  
 F17=0. 114030E+04

C  
 C  
 1211

OPT02890  
 OPT02900  
 OPT02910  
 OPT02920  
 OPT02930  
 OPT02940  
 OPT02950  
 OPT02960  
 OPT02970  
 OPT02980  
 OPT02990  
 OPT03000  
 OPT03010  
 OPT03020  
 OPT03030  
 OPT03040  
 OPT03050  
 OPT03060  
 OPT03070  
 OPT03080  
 OPT03090  
 OPT03100  
 OPT03110  
 OPT03120  
 OPT03130  
 OPT03140  
 OPT03150  
 OPT03160  
 OPT03170  
 OPT03180  
 OPT03190  
 OPT03200  
 OPT03210  
 OPT03220  
 OPT03230  
 OPT03240  
 OPT03250  
 OPT03260  
 OPT03270  
 OPT03280  
 OPT03290  
 OPT03300  
 OPT03310  
 OPT03320  
 OPT03330  
 OPT03340  
 OPT03350  
 OPT03360

```

F18=0. 236869E-24
F19=0. 936273E+02
F21=0. 104871E+04
F22=0. 194154E+00
F23=0. 990552E-08
F24=0. 843671E+04
F25=0. 310764E+01
F26=0. 000000E+00
F27=0. 439299E+04
F28=-. 119553E-24
F29=0. 495943E+03
F31=0. 708747E+04
F32=-. 167420E+00
F33=0. 109366E-07
F34=0. 884347E+04
F35=0. 414295E+01
F36=0. 000000E+00
F37=0. 646119E+04
F38=0. 623306E-24
F39=0. 659744E+03
GO TO 50

```

C  
1221

ANGM=-10 ANGO=60

```

ANGS=-30 ANGI=150
F11=0. 641881E+04
F12=-. 154288E+01
F13=0. 611579E-06
F14=-. 822035E+03
F15=0. 654956E+00
F16=0. 225757E+03
F17=0. 125410E+04
F18=-. 571492E-14
F19=0. 978107E+02
F110=-. 211747E+03
F21=-. 386867E+04
F22=0. 264005E+01
F23=0. 210582E-05
F24=0. 112568E+05
F25=0. 284393E+01
F26=0. 540771E+03
F27=0. 348582E+04
F28=-. 136893E-13
F29=0. 465491E+03
F210=0. 379750E+03
F31=0. 128335E+05
F32=-. 147761E+01
F33=0. 331809E-05
F34=0. 870688E+04
F35=0. 451243E+01

```

```

OPT03370
OPT03380
OPT03390
OPT03400
OPT03410
OPT03420
OPT03430
OPT03440
OPT03450
OPT03460
OPT03470
OPT03480
OPT03490
OPT03500
OPT03510
OPT03520
OPT03530
OPT03540
OPT03550
OPT03560
OPT03570
OPT03580
OPT03590
OPT03600
OPT03610
OPT03620
OPT03630
OPT03640
OPT03650
OPT03660
OPT03670
OPT03680
OPT03690
OPT03700
OPT03710
OPT03720
OPT03730
OPT03740
OPT03750
OPT03760
OPT03770
OPT03780
OPT03790
OPT03800
OPT03810
OPT03820
OPT03830
OPT03840

```

E36=-. 169267E+04  
 E37=0. 690758E+04  
 E38=0. 428491E-13  
 E39=0. 713575E+03  
 E310=-. 186004E+03  
 GO TO 50

C

1131

ANGS=0 ANGI=120 ANGM=-20 ANGO=60

E11=0. 811671E+04  
 E12=-. 196614E+01  
 E13=-. 333818E-07  
 E14=0. 188933E+04  
 E15=0. 4883956E-01  
 E16=0. 000000E+00  
 E17=-. 139207E+04  
 E18=0. 140756E-23  
 E19=0. 108239E+02  
 E110=-. 262390E+03  
 E21=-. 633743E+03  
 E22=0. 143016E+00  
 E23=-. 376307E-08  
 E24=0. 396744E+04  
 E25=0. 231410E+01  
 E26=0. 000000E+00  
 E27=0. 595382E+04  
 E28=0. 129334E-23  
 E29=0. 381170E+03  
 E210=0. 157266E+02  
 E31=-. 158742E+04  
 E32=0. 358846E+00  
 E33=-. 974890E-08  
 E34=0. 997180E+04  
 E35=0. 580637E+01  
 E36=0. 000000E+00  
 E37=0. 149255E+05  
 E38=0. 124027E-23  
 E39=0. 956405E+03  
 E310=0. 394602E+02  
 GO TO 50

C

1132

ANGS=0 ANGI=150 ANGM=-20 ANGO=60

E11=0. 759289E+04  
 E12=-. 195657E+01  
 E13=0. 870930E-08  
 E14=-. 874970E+03  
 E15=0. 965528E-01  
 E16=-. 807894E-23  
 E17=0. 867856E+03

OPT03850  
 OPT03860  
 OPT03870  
 OPT03880  
 OPT03890  
 OPT03900  
 OPT03910  
 OPT03920  
 OPT03930  
 OPT03940  
 OPT03950  
 OPT03960  
 OPT03970  
 OPT03980  
 OPT03990  
 OPT04000  
 OPT04010  
 OPT04020  
 OPT04030  
 OPT04040  
 OPT04050  
 OPT04060  
 OPT04070  
 OPT04080  
 OPT04090  
 OPT04100  
 OPT04110  
 OPT04120  
 OPT04130  
 OPT04140  
 OPT04150  
 OPT04160  
 OPT04170  
 OPT04180  
 OPT04190  
 OPT04200  
 OPT04210  
 OPT04220  
 OPT04230  
 OPT04240  
 OPT04250  
 OPT04260  
 OPT04270  
 OPT04280  
 OPT04290  
 OPT04300  
 OPT04310  
 OPT04320



F18=-. 131746E-24  
 F19=0. 171856E+02  
 F110=-. 331499E+03  
 F111=-. 472346E+04  
 F223=0. 633018E-07  
 F224=0. 348746E+04  
 F225=0. 317480E+01  
 F226=-. 549204E-24  
 F227=0. 967072E+04  
 F228=0. 226133E-23  
 F229=0. 5221581E+03  
 F230=0. 609826E+02  
 F331=-. 159741E+04  
 F332=-. 682238E-01  
 F333=0. 844020E-07  
 F334=0. 497586E+04  
 F335=0. 439744E+01  
 F336=-. 181799E-23  
 F337=0. 131649E+05  
 F338=0. 309438E-23  
 F339=0. 722932E+03  
 F3310=-. 120680E+02  
 GO TO 50

ANG=0 ANGI=150 ANGM=-30 ANGO=60

F11=0. 790298E+04  
 F112=-. 196635E+01  
 F113=0. 226082E-08  
 F114=0. 696921E+02  
 F115=0. 468845E-01  
 F116=-. 330942E-04  
 F117=0. 364668E+02  
 F118=0. 205154E-20  
 F119=0. 996732E+01  
 F1110=-. 262424E+03  
 F221=-. 821164E+03  
 F222=0. 181858E+00  
 F223=-. 929652E-08  
 F224=0. 915014E+04  
 F225=0. 320679E+01  
 F226=0. 104216E+04  
 F227=0. 467566E+04  
 F228=-. 623528E-13  
 F229=0. 522577E+03  
 F230=0. 199699E+02  
 F331=-. 117923E+04  
 F332=0. 248020E+00

OPT04330  
 OPT04340  
 OPT04350  
 OPT04360  
 OPT04370  
 OPT04380  
 OPT04390  
 OPT04400  
 OPT04410  
 OPT04420  
 OPT04430  
 OPT04440  
 OPT04450  
 OPT04460  
 OPT04470  
 OPT04480  
 OPT04490  
 OPT04500  
 OPT04510  
 OPT04520  
 OPT04530  
 OPT04540  
 OPT04550  
 OPT04560  
 OPT04570  
 OPT04580  
 OPT04590  
 OPT04600  
 OPT04610  
 OPT04620  
 OPT04630  
 OPT04640  
 OPT04650  
 OPT04660  
 OPT04670  
 OPT04680  
 OPT04690  
 OPT04700  
 OPT04710  
 OPT04720  
 OPT04730  
 OPT04740  
 OPT04750  
 OPT04760  
 OPT04770  
 OPT04780  
 OPT04790  
 OPT04800





$E_{26} = 0.127486E+04$   
 $E_{27} = 0.452784E+04$   
 $E_{28} = -0.208221E+03$   
 $E_{29} = 0.503443E+03$   
 $E_{30} = 0.233975E+02$   
 $E_{31} = -0.135845E+04$   
 $E_{32} = 0.283956E+00$   
 $E_{33} = 0.712038E+05$   
 $E_{34} = 0.120383E+01$   
 $E_{35} = 0.431677E+04$   
 $E_{36} = -0.293971E+04$   
 $E_{37} = 0.629262E+04$   
 $E_{38} = 0.480136E+03$   
 $E_{39} = 0.705153E+03$   
 $E_{40} = 0.311039E+02$   
 G TO 50

CALCULATE THE VALUE OF THE PITCH INPUT

IF (P.LT.1.5707) THEN  
P=8\*T\*K

```
ELSE  
P=1.570796327  
END IF
```

CALCULATE VARIABLE GAIN VALUES FROM X(K):

$$\begin{Bmatrix} C I = \cos \\ S I = \sin \end{Bmatrix} \begin{Bmatrix} X 3 \\ X 3 \end{Bmatrix} \begin{Bmatrix} K \\ K \end{Bmatrix}$$
$$\begin{aligned} \text{CM} &= \text{COS}(\text{X6}(\text{K})) \\ \text{SM} &= \text{SIN}(\text{X6}(\text{K})) \end{aligned}$$
$$\begin{pmatrix} C \\ S \end{pmatrix} = \begin{pmatrix} \cos \\ \sin \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix}$$
$$\begin{pmatrix} \cos(x) \\ \sin(x) \end{pmatrix}$$
$$Z=SM*SI$$
$$\begin{aligned} \text{JM} &= (0.07781 + 0.04949 * \text{CI}^{**2}) * 192 \\ \text{JO} &= (0.09318 + 0.02687 * \text{CM}^{**2} + 0.04428 * \text{CM}^{**2} * \text{SI}^{**2}) * 192 \end{aligned}$$
[illegible]

```

OPT05770
OPT05780
OPT05790
OPT05800
OPT05810
OPT05820
OPT05830
OPT05840
OPT05850
OPT05860
OPT05870
OPT05880
OPT05890
OPT05900
OPT05910
OPT05920
OPT05930
OPT05940
OPT05950
OPT05960
OPT05970
OPT05980
OPT05990
OPT06000
OPT06010
OPT06020
OPT06030
OPT06040
OPT06050
OPT06060
OPT06070
OPT06080
OPT06090
OPT06100
OPT06110
OPT06120
OPT06130
OPT06140
OPT06150
OPT06160
OPT06170
OPT06180
OPT06190
OPT06200
OPT06210
OPT06220
OPT06230
OPT06240

IF (TEST.EQ.1) R=PI
CALCULATE CONTROL INPUTS U1, U2 AND U3
  U1=-(F11*X1(K)+F12*X2(K)+F13*(X3(K)-R)+F14*X4(K)+F15*X5(K)
  2+F16*X6(K)+F17*X7(K)+F18*X8(K)+F19*X10(K)+F110*X11(K))
  U2=-(F21*X1(K)+F22*X2(K)+F23*(X3(K)-R)+F24*X4(K)+F25*X5(K)
  2+F26*X6(K)+F27*X7(K)+F28*X8(K)+F29*X10(K)+F210*X11(K))
  U3=-F31*X1(K)+F32*X2(K)+F33*(X3(K)-R)+F34*X4(K)+F35*X5(K)
  2+F36*X6(K)+F37*X7(K)+F38*X8(K)+F39*X10(K)+F310*X11(K))

CALCULATE THE K+1 VALUE OF EACH VARIABLE
*****
X1(K+1)=X1(K)+T*0.0678345*U1
X2(K+1)=X2(K)+T*GAIN*(-CS*X1(K)+CI*SS*X4(K)-SI*CM*SS*X7(K)-
  2Z*SO*SS*P)
X3(K+1)=X3(K)+T*(X1(K)-SM*X7(K)+CM*SO*P)
X4(K+1)=X4(K)+T*1/JM*U2
X5(K+1)=X5(K)+T*GAIN*(-CI*CS*X4(K)+CS*SI*CM*X7(K)-
  2SS*X1(K)+Z*SO*CS*P)
X6(K+1)=X6(K)+T*(X4(K)-CO*P)
X7(K+1)=X7(K)+T*1/JO*U3-SM/JO*U1)
X8(K+1)=X8(K)+T*(-CI*CM*X7(K)-SI*X4(K)-SO*CI*SM*P)
X9(K+1)=X9(K)+T*X7(K)
X10(K+1)=X10(K)+T*X5(K)
X11(K+1)=X11(K)+T*X2(K)

IF (K.EQ.2800) GO TO 777
CONTINUE

TTT=T*K
CALL FRTCMS('CLRSCRN ')
PRINT ', '
PRINT ', '
PRINT ', '
PRINT ', '
PRINT ', '
PRINT ', '
OPTIMAL FEEDBACK GAINS HAVE NOT BEEN COMPUTED '
FOR THE CURRENT GIMBAL ANGLES. NULLING SIN(AI) '

```



OPT06730  
OPT06740  
OPT06750

STOP  
END

C



# APPENDIX D TABULATED DATA

TIME (SEC)	UGSGF	UGSGF	VGSGF	INNER ANGLE	MIDDLE ANGLE
0.333333E-03	0.00000E+00	0.00000E+00	0.00000E+00	0.900001E+02	0.00000E+00
0.866666E-02	0.277583E-04	0.145522E-01	0.145522E-01	0.900143E+02	0.8346295E-02
0.170000E-01	0.103715E-02	-0.133000E+00	-0.133000E+00	0.900553E+02	0.332002E-01
0.253333E-01	0.62004E-02	-0.385341E+00	-0.385341E+00	0.901248E+02	0.742088E-01
0.336666E-01	0.219164E-01	-0.762853E+00	-0.762853E+00	0.9022211E+02	0.131484E+00
0.419999E-01	0.524359E-01	-0.126719E+01	-0.126719E+01	0.903448E+02	0.205019E+00
0.586666E-01	0.103713E+00	-0.189803E+01	-0.189803E+01	0.904958E+02	0.294804E+00
0.669999E-01	0.181147E+00	-0.265487E+01	-0.265487E+01	0.906741E+02	0.400826E+00
0.753333E-01	0.289998E+00	-0.353736E+01	-0.353736E+01	0.908799E+02	0.523072E+00
0.836666E-01	0.432543E+00	-0.454502E+01	-0.454502E+01	0.9111130E+02	0.661523E+00
0.919999E-01	0.622578E+00	-0.567730E+01	-0.567730E+01	0.913735E+02	0.81615258E+00
0.100333E+00	0.856513E+01	-0.693339E+01	-0.693339E+01	0.9166135E+02	0.986950E+00
0.108667E+00	0.114233E+01	-0.831333E+01	-0.831333E+01	0.919766E+02	0.1173887E+01
0.117000E+00	0.148502E+01	-0.981594E+01	-0.981594E+01	0.9233193E+02	0.137688E+01
0.125333E+00	0.188966E+01	-0.114406E+02	-0.114406E+02	0.926894E+02	0.159594E+01
0.133667E+00	0.239042E+01	-0.131867E+02	-0.131867E+02	0.930869E+02	0.183100E+01
0.142000E+00	0.352500E+01	-0.150534E+02	-0.150534E+02	0.9335120E+02	0.208201E+01
0.150333E+00	0.422702E+01	-0.170402E+02	-0.170402E+02	0.936646E+02	0.234891E+01
0.158666E+00	0.509571E+01	-0.191464E+02	-0.191464E+02	0.944448E+02	0.263162E+01
0.167000E+00	0.587201E+01	-0.233713E+02	-0.233713E+02	0.949582E+02	0.293006E+01
0.175333E+00	0.687933E+01	-0.267174E+02	-0.267174E+02	0.9548516E+02	0.3234414E+01
0.183666E+00	0.794933E+01	-0.287511E+02	-0.287511E+02	0.9605428E+02	0.357375E+01
0.192000E+00	0.913241E+01	-0.314444E+02	-0.314444E+02	0.9666428E+02	0.391879E+01
0.200333E+00	0.1042209E+02	-0.349000E+02	-0.349000E+02	0.972620E+02	0.427913E+01
0.208666E+00	0.116269E+02	-0.216803E+02	-0.216803E+02	0.9790655E+02	0.4652295E+01
0.217000E+00	0.668379E+02	-0.204485E+02	-0.204485E+02	0.985557E+02	0.502813E+01
0.225333E+00	0.406588E+01	-0.220079E+02	-0.220079E+02	0.992014E+02	0.540227E+01
0.233666E+00	0.7345961E+01	-0.224933E+02	-0.224933E+02	0.998553E+02	0.5775229E+01
0.242000E+00	0.1592441E-02	-0.223943E+02	-0.223943E+02	0.100507E+03	0.614706E+01
0.250333E+00	0.628444E+00	-0.221652E+02	-0.221652E+02	0.101159E+03	0.651756E+01
0.258666E+00	0.2701733E+00	-0.221838E+02	-0.221838E+02	0.101813E+03	0.688676E+01
0.267000E+00	0.301474E+00	-0.221533E+02	-0.221533E+02	0.102467E+03	0.725461E+01
0.275333E+00	0.332833E+00	-0.220001E+02	-0.220001E+02	0.103122E+03	0.762105E+01
0.283666E+00	0.300393E+00	-0.141415E+02	-0.141415E+02	0.1037738E+03	0.798606E+01
0.292000E+00	0.1529248E+01	-0.6299700E+02	-0.6299700E+02	0.104436E+03	0.834959E+01
0.300333E+00	0.1282248E+00	-0.390406E+02	-0.390406E+02	0.105095E+03	0.8711255E+01
0.308666E+00	0.173000E+00	-0.1999967E+02	-0.1999967E+02	0.105755E+03	0.906673E+01
0.317000E+00	0.219891E+00	-0.180456E+02	-0.180456E+02	0.106415E+03	0.942558E+01
0.325333E+00	0.2525724E+00	-0.180456E+02	-0.180456E+02	0.107077E+03	0.977518E+01
0.334000E+00	0.2216717E+00	-0.180456E+02	-0.180456E+02	0.107741E+03	0.101213E+02
0.342333E+00	0.2216717E+00	-0.180456E+02	-0.180456E+02	0.108405E+03	0.104636E+02
0.350666E+00	0.2216717E+00	-0.180456E+02	-0.180456E+02	0.1090737E+03	0.108017E+02
0.358666E+00	0.2216717E+00	-0.180456E+02	-0.180456E+02	0.109737E+03	0.111463E+02
0.367000E+00	0.2216717E+00	-0.180456E+02	-0.180456E+02	0.110405E+03	0.1146927E+02
0.375333E+00	0.2216717E+00	-0.180456E+02	-0.180456E+02	0.111075E+03	0.1179227E+02
0.383666E+00	0.2216717E+00	-0.180456E+02	-0.180456E+02	0.111745E+03	0.121154E+02
0.392000E+00	0.2216717E+00	-0.180456E+02	-0.180456E+02	0.112417E+03	0.124343E+02



[illegible]

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